

5. The dimensions of the column space and the null space for an $m \times n$ matrix are equal.

A) True

B) False

(Not true in general. See the rank theorem for what is true.)

6. U is an $m \times n$ matrix with orthonormal columns if and only if $U^T U = I$.

A) True

B) False

(Thm. 6 / p. 345)
(Thm 7 / p. 345)

7. If U is an $m \times n$ matrix with orthonormal columns, and $\mathbf{x} \in \mathbb{R}^n$, then $\|U\mathbf{x}\| = \|\mathbf{x}\|$.

A) True

B) False

8. If \mathbf{v} and \mathbf{w} are eigenvectors of A for the same eigenvalue λ , then \mathbf{v} and \mathbf{w} are linearly independent.

A) True

B) False

Notice, for example, that if \vec{v} is an eigenvector ($A\vec{v} = \lambda\vec{v}$)
the $2\vec{v}$ is also an eigenvector for λ ,

9. If A is a matrix with real entries and $3 - 4i$ is an eigenvalue of A then $3 + 4i$ is also an eigenvalue.

A) True

B) False

($A(2\vec{v}) = \lambda(2\vec{v})$)

10. Every invertible matrix is diagonalizable.

A) True

B) False

[Try some examples.]

11. If $u \in W$, where W is a subspace of \mathbb{R}^n , the $proj_W u = u$.

A) True

B) False

12. Suppose that A is a 6×6 matrix with characteristic equation $(\lambda - 3)(\lambda + 7)(\lambda^2 - 3)(\lambda^2 - 3\lambda - 1)$. Then A is diagonalizable.

A) True

B) False

6 distinct eigenvalues: $\lambda_1 = 3, \lambda_2 = -7, \lambda_3 = -\sqrt{3}, \lambda_4 = \sqrt{3},$
 $\lambda_5, \lambda_6 = \frac{3 \pm \sqrt{13}}{2}$ (Thm 6 p. 286)