Part II: Multiple Choice Select the best answer for each question.

13. If A and B are 4×4 matrices, $\det(A) = 7$ and $\det(B) = 8$, what is $\det((2A)B^{-1})$?

A) 1 B) -11 C) 14 D) 12 E) -12 F) 56 G) -112 H) none of the above

$\det((2A)B^{-1}) = 2^4 \cdot 7 \cdot \frac{1}{8} = 14$

14. Consider the system of equations

$$5x_1 - 7x_2 = 12$$

$$25x_1 + hx_2 = k.$$

Give conditions on h and k such that the system has *many* solutions.

A) $h = -35$ B) $h = -35$ and $k = 60$ C) $h \neq -35$ and $k = 60$
D) $h = -35$ and $k \neq 60$ E) $h \neq -35$ and $k \neq 60$ F) None of the above.

15. Let A be an $m \times n$ matrix and suppose the reduced echelon form of A has four rows of zeros. What is the rank of A?

A) m B) n C) n - m D) n - 4 E) m - 4 = # of pivots
F) m + n G) m + n +2 H) none of the above

16. Let $\mathbf{x} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$ (in coordinates of the standard basis for $\mathbb{R}^2$) and let $\mathcal{B} = \{\begin{bmatrix} 2 \\ 1 \end{bmatrix}, \begin{bmatrix} -1 \\ 3 \end{bmatrix}\}$ be an alternate basis. Find $[\mathbf{x}]_{\mathcal{B}}$.

A) $\begin{bmatrix} -1 \\ 11 \end{bmatrix}$ B) $\begin{bmatrix} -1 \\ \frac{1}{2} \end{bmatrix}$ C) $\begin{bmatrix} -1 \\ 1 \end{bmatrix}$ D) $\begin{bmatrix} \frac{10}{3} \\ \frac{1}{3} \end{bmatrix}$

E) none of the above Specifically, $\begin{bmatrix} 1 \\ 0 \end{bmatrix} = [x]_{\mathcal{B}}$. (Notice that $\begin{bmatrix} 2 \\ 1 \end{bmatrix}$ is a basis element.)

17. For a discrete dynamical system given by $\mathbf{x}_{k+1} = A\mathbf{x}_k$, the trajectories all spiral in towards the origin. Which of the following might be the eigenvalues of the 2×2 matrix A?

A) 1, -2 B) 0.5, -2 C) 0.5, .75 D) 3, 4
E) 0.9+0.2i, 0.9-0.2i F) 0.8+0.7i, 0.8-0.7i G) none of the above

Spiral = complex eigenvalues, towards origin = $\|z\| < 1$ $\left(\sqrt{.9^2 + .2^2} = \sqrt{.85} < 1\right)$.