18. Find a least squares solution to the inconsistent equation $Ax = b$ where

$A = \begin{bmatrix} 1 & 5 \\ 2 & -2 \\ -1 & 1 \end{bmatrix}$ and $\mathbf{b} = \begin{bmatrix} 3 \\ 2 \\ 5 \end{bmatrix}$

$A^TA = \begin{bmatrix} 1 & 2 & -1 \\ 5 & -2 & 1 \end{bmatrix}\begin{bmatrix} 1 & 5 \\ 2 & -2 \\ -1 & 1 \end{bmatrix} = \begin{bmatrix} 6 & 0 \\ 0 & 30 \end{bmatrix}$ $(A^TA)^{-1} = \frac{1}{180}\begin{bmatrix} 30 & 0 \\ 0 & 6 \end{bmatrix}$

A) $\begin{bmatrix} 17 \\ -12 \end{bmatrix}$ B) $\begin{bmatrix} \frac{1}{2} \\ \frac{1}{3} \end{bmatrix}$ C) $\begin{bmatrix} \frac{1}{3} \\ \frac{8}{15} \end{bmatrix}$ D) $\begin{bmatrix} \frac{1}{9} \\ \frac{1}{3} \end{bmatrix}$ E) $\begin{bmatrix} \frac{2}{3} \\ \frac{11}{13} \end{bmatrix}$

$A^Tb = \begin{bmatrix} 2 \\ 16 \end{bmatrix}$

F) $\begin{bmatrix} \frac{1}{2} \\ \frac{1}{4} \end{bmatrix}$ G) none of the above

$\hat{x} = (A^TA)^{-1}A^Tb = \frac{1}{180}\begin{bmatrix} 30 & 0 \\ 0 & 6 \end{bmatrix}\begin{bmatrix} 2 \\ 16 \end{bmatrix} = \frac{1}{180}\begin{bmatrix} 60 \\ 96 \end{bmatrix} = \begin{bmatrix} \frac{1}{3} \\ \frac{8}{15} \end{bmatrix}$

19. Let $V = P_2$, the vector space of degree two polynomials. Define an inner product on V by

$$f \cdot g = \int_0^1 f(x)g(x)dx.$$

In this inner product space are $p(x) = x + 4$ and $q(x) = x^2$ orthogonal?

A) Yes B) No

$p \cdot q = \int_0^1 (x+4)x^2\,dx = \int_0^1 x^3 + 4x^2\,dx = \left[\frac{1}{4}x^4 + \frac{4}{3}x^3\right]_0^1$

$= \frac{1}{4} + \frac{4}{3} = \frac{19}{12} \neq 0$

20. If $T : P_1 \to P_1$ is a transformation on linear functions such that $T(1 + x) = 3 + 2x$ and $T(2 - 3x) = 11 - x$, find $T(3 + 2x)$.

A) 75-30x B) 10+5x C) 12-3x D) 12+42x E) 12x F) None of the above

$3(T(1) + T(x)) = (3 + 2x)$ $\rightarrow 5T(1) = 20 + 5x$

$2T(1) - 3T(x) = 11 - x$ $\leftarrow T(1) = 4 + x$

$2(4 + x) - 3T(x) = 11 - x$ $\rightarrow -3T(x) = 3 - 3x$

$T(x) = x - 1$

$T(3 + 2x) = 3(4 + x) + 2(x - 1) = 10 + 5x$

21. Let $V = P_2$, the vector space of degree two polynomials. Define an inner product on V for $f(x) = ax^2 + bx + c$ and $g(x) = rx^2 + sx + t$ by

$$f \cdot g = ar + bs + ct.$$

In this inner product space are $p(x) = x + 4$ and $q(x) = x^2$ orthogonal?

A) Yes B) No

$p \cdot q = 0 \cdot 1 + 1 \cdot 0 + 4 \cdot 0 = 0$