22. Let $V = C^\infty(\mathbb{R})$, the vector space of smooth functions. By the Constant Multiplier and Sum rules for derivative, we know that the derivative is a linear transformation $D : V \to V$. Which of the following functions is **not** an eigenvector for D?

A) $f(x) = e^x$ B) $f(x) = e^{2x}$ C) $f(x) = 3e^x$
D) $f(x) = 3e^{2x}$ E) $f(x) = xe^{2x}$ F) $f(x) = 2^x$
G) none of the above

$$D(xe^{2x}) = e^{2x} + 2xe^{2x} \neq \lambda \cdot xe^{2x}$$

23. Which of the following conditions is *not* equivalent to an $n \times n$ matrix A being invertible?

A) The number 0 is an eigenvalue of A

B) $\det(A) \neq 0$

C) ColA$=\mathbb{R}^n$

D) rank$A$=n

E) dim Nul A=0

F) A^T is an invertible matrix

G) A has n pivot positions

H) The columns of A span $\mathbb{R}^n$

I) A is row equivalent to the identity matrix

J) The columns of A are linearly independent

K) none of the above

See Invertible Matrix Theorem (Version 3)

24. Which of the following vectors is *not* in the eigenspace of $A = \begin{bmatrix} -1 & 0 & 1 \\ 3 & 0 & -3 \\ 1 & 0 & -1 \end{bmatrix}$

A) $\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$ B) $\begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$ C) $\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$ D) $\begin{bmatrix} 3 \\ 0 \\ 3 \end{bmatrix}$ E) $\begin{bmatrix} -1 \\ 3 \\ 1 \end{bmatrix}$

Note that any multiple of an eigenvector is still an eigenvector.

F) All of the above are in the eigenspace

G)More than one of the above vectors is not in the eigenspace

$\det A = 0 \Rightarrow -\lambda^2(\lambda+2) = 0 \Rightarrow \lambda = 0$ OR $\lambda = -2$

$\lambda = 0$ (mult. 2) $\rightarrow \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$ $\lambda = -2 \rightarrow \begin{bmatrix} -1 \\ 3 \\ 1 \end{bmatrix}$