Math 309 Practice Exam 1

Part I

1. b
2. f
3. e
4. e
5. g
6. a
7. c
8. c
9. f
10. a

Part II

1. (a) $A_1 = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$, (b) $A_2 = \begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}$, (c) $A_T = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} = (a) \begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} = A_2 A_1$

2. (a) TRUE Proof: $AC = AD$ ($+ A^{-1}$ exists) $\Rightarrow A^{-1}AC = A^{-1}AD$
$\Rightarrow IC = ID$
$\Rightarrow C = D$

(b) False

(c) False

3. x_3 free, $\begin{bmatrix} x_3 - 70 \\ x_3 - 30 \\ x_3 \\ 70 \end{bmatrix}$ (or $x_1 = x_3 - 70,\ x_2 = x_3 - 30,\ x_4 = 70$)

4. (a) $A^{-1} = \begin{bmatrix} -\frac{3}{23} & \frac{9}{23} & -\frac{1}{46} \\ -\frac{8}{23} & \frac{1}{23} & \frac{5}{46} \\ \frac{9}{23} & -\frac{4}{23} & \frac{3}{46} \end{bmatrix}$

(b) Not invertible
(columns are dependent, for example)