Part III: Short Answer(20)

You do not need to show any work for this part, just answer each question or fill in the blank.

17. The matrix $A = \begin{bmatrix} 12 & -2 & 0 & 0 & 3 \\ 2 & -5 & -3 & -2 & 6 \\ 0 & 5 & 15 & 10 & 0 \\ 2 & 6 & 18 & 8 & 6 \end{bmatrix}$ reduces to $\begin{bmatrix} 1 & -2 & 0 & 0 & 3 \\ 0 & 1 & 3 & 2 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$. $\sim \begin{bmatrix} 1 & 0 & 0 & -2 & 3 \\ 0 & 1 & 0 & -1 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$

Need for g.

(a) row(A) is a subspace of $\mathbb{R}^n$ where $n =$ 5

(b) Col(A) is a subspace of $\mathbb{R}^n$ where $n =$ 4

(c) Nul(A) is a subspace of $\mathbb{R}^n$ where $n =$ 5

(d) rank(A)= 3

Give a basis for:

(e) row(A)

$$\mathcal{B} = \{[1\ -2\ 0\ 0\ 3], [0\ 1\ 3\ 2\ 0], [0\ 0\ 1\ 1\ 0]\}$$

(f) Col(A)

$$\mathcal{B} = \left\{ \begin{bmatrix} 12 \\ 2 \\ 0 \\ 2 \end{bmatrix}, \begin{bmatrix} -2 \\ 5 \\ 5 \\ 6 \end{bmatrix}, \begin{bmatrix} 0 \\ -3 \\ 15 \\ 18 \end{bmatrix} \right\}$$

(g) Nul(A) $\mathcal{B} = \left\{ \begin{bmatrix} 2 \\ 1 \\ -1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -3 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \right\}$ (Should have dimension 2 by Rank Theorem)

$x_1 = 2x_4 - 3x_5$
$x_2 = x_4$
$x_3 = -x_4$

$$\longrightarrow \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = x_4 \begin{bmatrix} 2 \\ 1 \\ -1 \\ 1 \\ 0 \end{bmatrix} + x_5 \begin{bmatrix} -3 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$