MATH 4121 INFORMATION: SPRING, 2015

INSTRUCT0R: Edward N. Wilson, Office 109A, Cupples I,
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PRE-REQUISITES: Math 4111 or its equivalent is a firm pre-requisite. Highly useful but not essential is some knowledge of probability theory.

CLASSES: The class will meet Tuesdays and Thursdays from 11:30 a.m. until 1:00 p.m in Room 199, Cupples. I

OFFICE HOURS: The tentative plan is to have office hours Tuesdays and Wednesdays from 2:00 p.m. until 4:00 p.m.. But this plan will be revised if it's not convenient for students. Also, students are welcome to send an e-mail to the instructor asking to meet him in his office outside of office hours. Such e-mails should be sent at least one full day before the requested meeting time.

TEXTBOOK: Robert Bartle, The Elements of Integration and Lebesgue Measure, Wiley Classics Paperback Edition, 1995. As the title suggests, this is an introductory text which is fairly readable and has reasonable exercises. It covers most of the key aspects of the subject without added frills.

HIGHLY RECOMMENDED SUPPLEMENTARY BOOK: Gerald Folland, Real Analysis: Modern Techniques and Their Applications, Second Edition, Wiley-Interscience Series, 1999. Folland's book is arguably the best single graduate level real analysis text now available. In addition to covering everything in Bartle's book,
Folland goes on to cover in depth many things not mentioned by Bartle: Fourier analysis, the Lebesgue Differention Theory, The Change of Variable Theorem for Lebesgue integrals, Distribution Theory, Sobolov spaces, etc. Folland also has excellent short chapters on logic and the foundations of mathematics, the part of topology especially relevant to analysis, analysis on manifolds, functional analysis, and probability. Literally, Folland's book in a personal library takes the place of 5 or 6 books each of which only covers a few of these topics. But, Folland's writing style is terse (meaning he leaves out MANY details), his problems are sometimes very difficult and the hints aren't all that helpful, and there are quite a few unfortunate typographical errors in his book which can be very disconcerting to those studying the material for the first time. These are the reasons why Folland's book wasn't selected as the course textbook. But for those who expect to take more graduate level courses in analysis and/or probability, Folland's book is very highly recommended; as a reference guide down the road, it will be much more valuable than Bartle's book.

GRADING: Grades for the course will be based on weekly homework assignments (40%), a midterm in-class exam (20%), and the final exam (40%). During the last week of the semester, pre-final class averages based on homework and the midterm will be compiled. Those with high pre-final averages will be excused from the final and will receive A's for the course.

ACADEMIC INTEGRITY. Incidents of cheating on either the mid-term exam or the final exam will be reported to the Arts and Sciences Integrity Committee for its adjudication. Appearing before this Committee is a very unpleasant ordeal. If the Committee finds that cheating took place, it frequently dictates to the instructor that the guilty student (or students) receive the grade of No Credit for the semester. PLEASE DON'T CHEAT on the exams.

Homework is an entirely different matter. It's almost impossible to learn measure theory without working out problems and writing up the solutions. Students are welcome to talk among themselves or with the instructor about ways to approach the homework problems. BUT, EVERY STUDENT MUST WRITE UP HIS/HER HOMEWORK SOLUTIONS. When the grader has reason to believe that one student solved a homework problem and others merely copied off this solution, there will be a severe deduction in credit awarded., e.g., with 4 virtually identical solutions and no way to distinguish the problem solver from those who copied, all 4 students will receive at most ¼ of the maximum credit available for the assignment. Also, when the grader has reason to believe that a student copied off a solution available on-line, little or no credit will be awarded.

OVERVIEW OF THE COURSE. Math 4121 is a one-semester course on Lebesgue measure and integration. This is vital material for everyone interested in probability theory and its applications as well as those hoping to do research in mathematical analysis. It's also part of the "common core" for graduate programs in mathematics and its applications.

As sketched in Bartle's Introductory Chapter (which everyone should read), a measure on a class S of subsets of some fixed set X is a function m assigning a non-negative number to each member of S. We think of these numbers as measuring size or "volume". We insist that S is closed under countable unions and set complements and that the size of a countable disjoint union is the sum of the sizes of the constituent parts of the union. When f is a non-negative valued function on X for which the inverse image under f of any interval is in S, one can define the integral of with respect to the measure m as a limit of Lebesgue sums and then go on to prove a host of theorems about these integrals.

All of the above is very general and very abstract. But this high degree of abstraction is precisely what's used in the development of probability theory. Indeed, a general measure on subsets of X is a probability measure if it assigns to X the size 1. In probability theory, functions on X with the above-mentioned inverse image property are called random variables and their integrals are called expected values or means.

One-dimensional Lebesgue measure on $\boldsymbol{R}$ is an extension of the length function on intervals and n-dimensional Lebesgue measure on $\boldsymbol{R}^n$ is an extension of the Euclidean n-dimensional volume for rectangles. When a function on $\boldsymbol{R}^n$ is Riemann integrable, it's also Lebesgue integrable and the value of the Riemann integral coincides with the value of the Lebesgue integral. But there are many more Lebesgue integrable functions than Riemann integrable functions. The limit theorems for Riemann integration have hard to verify hypotheses. In contrast, the limit theorems for Lebesgue integrals have hypotheses which are usually easily verifiable. Because of this, Riemann integration has been replaced by Lebesgue integration for researchers in the analysis of functions on $\boldsymbol{R}^n$

OUTLINE OF THE ORDER OF TOPICS

1. We'll begin with the formal definitions of measures (Chapter 2 in the text) and then go on to rigorously construct Lebesgue measure and many other measures (Chapter 9 in the text). We'll then discuss the special properties of Lebesgue measure (parts of Chapters 11-17).

2. The basics of general integration theory (Chapters 3-5 and 10).

3. L^P spaces and modes of convergence (Chapters 6 and 7).

4. Radon-Nikodym derivatives (Chapter 8)and the Lebesgue Differentiation Theroem (a far-reaching generalization of the Fundamental Theorem of Calculus not mentioned in the text)

5. (As time permits) Harmonic analysis including properties of convolutions, Fourier series, and Fourier integrals. These topics are also not mentioned in the text but are of huge importance for applications in engineering and physics.

Link for Homework Assigments and Notes