

In 2015, Brosnan and Chow proved the Shareshian--Wachs conjecture, which links an open conjecture of Stanley and Stembridge to the geometry of Hessenberg varieties through Tymoczko's permutation group action on the cohomology ring of regular semisimple Hessenberg varieties. This talk will give a brief overview of that story and present a new set of linear relations satisfied by the multiplicities of certain permutation representations in Tymoczko's representation. As an application of these results, we prove an inductive formula for the multiplicity coefficients corresponding to partitions with a maximal number of parts. This talk is based on joint work with M. Harada.