The Brieskorn and Orlik-Solomon theorems state that the cohomology algebra of the complement of a complex hyperplane arrangement is isomorphic to the Orlik-Solomon algebra of its intersection lattice. In this talk, I will show that homotopy sphere arrangements arising as homotopy colimits of diagrams of spaces on a geometric lattice can be embedded into topological spheres when the codimension is greater than or equal to two. This gives a formula for the cohomology groups of the complements of these arrangements and raises a question about the relationship between the cohomology algebras of these complements and the Orlik-Solomon algebra of the given lattice. This is joint work with Alex Engström.