Homework set 9 - due 11/20/26

Math 3010 – Renato Feres

Note 1. Work on all the problems listed below, but only those marked with an $(*)$ need to be turned in.
Note 2. The problems marked with $(*)$ are all worth the same number of points.

$\diamond$

1. **Read Chapter 14 of Liebeck's textbook up to (including) Proposition 14.1.**
2. **Chapter 14, Exercise 1** $(*)$
   (a) Find $3^{301} \pmod{11}$, $5^{110} \pmod{13}$ and $7^{1388} \pmod{127}$.
   (b) Show that $n^7 - n$ is divisible by 42 for all positive integers n.
3. **Read Chapter 18 of Liebeck's textbook.**
4. **Chapter 18, Exercise 1** $(*)$ Which of the following relations are equivalence relations on the given set S?
   (a) $S = \mathbb{R}$, and $a \sim b \Leftrightarrow a = b$ or $a = -b$.
   (b) $S = \mathbb{Z}$, and $a \sim b \Leftrightarrow ab = 0$.
   (c) $S = \mathbb{R}$, and $a \sim b \Leftrightarrow a^2 + a = b^2 + b$.
   (d) S is the set of all people in the world, and $a \sim b$ means a lives within 100 miles of b.
   (e) S is the set of all points in the plane, and $a \sim b$ means a and b are the same distance from the origin.
   (f) $S = \mathbb{N}$, and $a \sim b \Leftrightarrow ab$ is a square.
   (g) $S = \{1, 2, 3\}$, and $a \sim b \Leftrightarrow a = 1$ or $b = 1$.
   (h) $S = \mathbb{R} \times \mathbb{R}$, and $(x, y) \sim (a, b) \Leftrightarrow x^2 + y^2 = a^2 + b^2$.
5. **Chapter 18, Exercise 2** $(*)$ For those relations in the previous exercise that are equivalence relations, describe the equivalence classes.
6. **Chapter 18, Exercise 7** $(*)$ Let $\sim$ be an equivalence relation on $\mathbb{Z}$ with the property that for all $m \in \mathbb{Z}$ we have $m \sim m + 5$ and also $m \sim m + 8$. Prove that $m \sim n$ for all $m, n \in \mathbb{Z}$.
7. **Read Chapter 16 of Liebeck's textbook.**
8. **Chapter 16, Exercise 8** $(*)$
   (a) Prove that
   $$\binom{n+1}{r} = \binom{n}{r} + \binom{n}{r-1}.$$
   (b) Prove that for any positive integer n,
   $$3^n = \sum_{k=0}^{n} \binom{n}{k} 2^k.$$