Note 1. Work on all the problems listed below, but only those marked with an (∗) need to be turned in.
Note 2. The problems marked with (∗) are all worth the same number of points.
Note 3. Chapters and exercise numbers refer to the 4th edition of Liebeck's text.

1. Read Chapter 14 of Liebeck's textbook up to (including) Proposition 14.1.

2. Chapter 14, Exercise 1 (∗)
   (a) Find \( 3^{301} \pmod{11} \), \( 5^{110} \pmod{13} \) and \( 7^{1388} \pmod{127} \).
   (b) Show that \( n^7 - n \) is divisible by 42 for all positive integers \( n \).

3. Read Chapter 18 of Liebeck's textbook.

4. Chapter 18, Exercise 1 (∗) Which of the following relations are equivalence relations on the given set \( S \)?
   (a) \( S = \mathbb{R} \), and \( a \sim b \iff a = b \) or \( a = -b \).
   (b) \( S = \mathbb{Z} \), and \( a \sim b \iff ab = 0 \).
   (c) \( S = \mathbb{R} \), and \( a \sim b \iff a^2 + a = b^2 + b \).
   (d) \( S \) is the set of all people in the world, and \( a \sim b \) means \( a \) lives within 100 miles of \( b \).
   (e) \( S \) is the set of all points in the plane, and \( a \sim b \) means \( a \) and \( b \) are the same distance from the origin.
   (f) \( S = \mathbb{N} \), and \( a \sim b \iff ab \) is a square.
   (g) \( S = \{1, 2, 3\} \), and \( a \sim b \iff a = 1 \) or \( b = 1 \).
   (h) \( S = \mathbb{R} \times \mathbb{R} \), and \( (x, y) \sim (a, b) \iff x^2 + y^2 = a^2 + b^2 \).

5. Chapter 18, Exercise 2 (∗) For those relations in the previous exercise that are equivalence relations, describe the equivalence classes.

6. Chapter 18, Exercise 7 (∗) Let \( \sim \) be an equivalence relation on \( \mathbb{Z} \) with the property that for all \( m \in \mathbb{Z} \) we have \( m \sim m + 5 \) and also \( m \sim m + 8 \). Prove that \( m \sim n \) for all \( m, n \in \mathbb{Z} \).

7. Read Chapter 16 of Liebeck's textbook.

8. Chapter 16, Exercise 8 (∗)
   (a) Prove that
   \[
   \binom{n + 1}{r} = \binom{n}{r} + \binom{n}{r - 1}
   \]
(b) Prove that for any positive integer \( n \),

\[
3^n = \sum_{k=0}^{n} \binom{n}{k} 2^k.
\]