Homework set 7 - due 04/01/2022

Math 495 – Renato Feres

Problems

The problems to be submitted to Crowdmark are 2, 4 and 5. (As always, work on all of them.)

1. (Text, Exercise 6.1, page 258.) Let $(N_t)_{t\geq 0}$ be a Poisson process with parameter $\lambda = 1.5$. Find the following:
   (a) $P(N_1 = 2, N_4 = 6)$
   (b) $P(N_4 = 6 | N_1 = 2)$
   (c) $P(N_1 = 2 | N_4 = 6)$.
2. (Text, Exercise 6.2, page 258.) Let $(N_t)_{t\geq 0}$ be a Poisson process with parameter $\lambda = 2$. Find the following:
   (a) $E(N_3 N_4)$
   (b) $E(X_3 X_4)$
   (c) $E(S_3 S_4)$
3. (Text, Exercise 6.3, page 258.) Calls are received at a company call center according to a Poisson process at the rate of five calls per minute.
   (a) Find the probability that no call occurs over a 30-second period.
   (b) Find the probability that exactly four calls occur in the first minute, and six calls occur in the second minute.
   (c) Find the probability that 25 calls are received in the first 5 minutes and six of those calls occur in the first minute.
4. (Text, Exercise 6.4, page 258.) Starting at 9 a.m., patients arrive at a doctor's office according to a Poisson process. On average, three patients arrive every hour.
   (a) Find the probability that at least two patients arrive by 9 : 30 a.m.
   (b) Find the probability that 10 patients arrive by noon and eight of them come to the office before 11 a.m.
   (c) If six patients arrive by 10 a.m., find the probability that only one patient arrives by 9 : 15 a.m.
5. (Text, Exercise 6.6, page 259.) Occurrences of landfalling hurricanes during an El Niño event are modeled as a Poisson process in Bove et al. (Effects of El Niño on U.S. landfalling hurricanes, revisited. *Bulletin of the American Meteorological Society*, 79(11):2477-2482, 1998.) The authors assert that "During an El Niño year, the probability of two or more hurricanes making landfall in the United States is 28%." Find the rate of the Poisson process.

6. (Text, Exercise 6.7, page 259.) Ben, Max, and Yolanda are at the front of three separate lines in the cafeteria waiting to be served. The serving times for the three lines follow independent Poisson processes with respective parameters $1, 2$, and 3.

   (a) Find the probability that Yolanda is served first.

   (b) Find the probability that Ben is served before Yolanda.

   (c) Find the expected waiting time for the first person served.