Homework set 8 - due 04/08/22

Math 495 – Renato Feres

Submit to Crowdmark your solutions to problems 2, 3, and 4.

Problems

1. (Text, Exercise 7.2, page 313.) A Markov chain on $\{1,2,3,4\}$ has nonzero transition rates

$$q_{1,2} = q_{2,3} = q_{31} = q_{41} = 1 \text{ and } q_{14} = q_{32} = q_{34} = q_{43} = 2.$$

   (a) Exhibit the (infinitesimal) generator, holding time parameters, and the transition matrix for the embedded Markov chain.

   (b) If the chain is at state 1, how long on average will it take before moving to a new state?

   (c) If the chain is at state 3, how long on average will it take before moving to state 4? (See page 288 under heading "Absorbing states.")

   (d) Over the long term, what proportion of visits will be to state 2?

2. (Text, Exercise 7.4, page 313.) During lunch hour, customers arrive at a fast-food restaurant at the rate of 120 customers per hour. The restaurant has one line, with three workers taking food orders at independent service stations. Each worker takes an exponentially distributed amount of time—on average 1 minute—to service a customer. Let X_t denote the number of customers in the restaurant (in line and being served) at time t. The process $(X_t)_{t\geq 0}$ is a continuous-time Markov chain. Exhibit the generator matrix.

3. (Text, Exercise 7.6, page 313.) A Markov chain $(X_t)_{t\geq 0}$ on $\{1,2,3,4\}$ has generator matrix

$$Q = \begin{pmatrix} -2 & 1 & 1 & 0 \\ 1 & -3 & 1 & 1 \\ 2 & 2 & -4 & 0 \\ 1 & 2 & 3 & -6 \end{pmatrix}.$$

   Use technology as needed for the following:

   (a) Find the long-term proportion of time that the chain visits state 1.

   (b) For the chain started in state 2, find the long-term probability that the chain visits state 3.

   (c) Find $P(X_1 = 3|X_0 = 1)$.

   (d) Find $P(X_5 = 1, X_2 = 4|X_1 = 3)$.

4. (Text, Exercise 7.14, page 315.) For the Markov chain with transition rate graph shown in the figure, find

   (a) the generator matrix,

(b) the stationary distribution of the continuous-time Markov chain,

(c) the transition matrix of the embedded chain,

(d) the stationary distribution of the embedded chain.

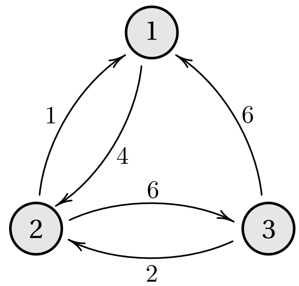

Figure 1: Diagram for Exercise 4.