Homework set 2 - due 09/18/22

Math 5047

1. Read Chapter 1 of do Carmo's Riemannian Geometry.
2. (do Carmo, page 45, Exercise 1.)
   (a) Prove that the antipodal mapping $A : S^n \to S^n$ given by $A(p) = -p$ is an isometry of S^n.
   (b) Use this fact to introduce a Riemannian metric on the real projective space $P^n(\mathbb{R})$ such that the natural projection $\pi : S^n \to P^n(\mathbb{R})$ is a local isometry.
3. (do Carmo, page 46, Exercise 3.) Obtain an isometric immersion of the flat torus $\mathbb{T}^n$ into $\mathbb{R}^{2n}$.
4. Let $\eta(\mathbf{x})$ be a positive function defined on an open subset $\mathscr{U}$ in $\mathbb{R}^n$ and define the Riemannian metric on $\mathscr{U}$

   $$\langle u, v\rangle_x := \eta^2(x) u \cdot v.$$

   We say that the metric is conformally Euclidean.
   (a) Express the Riemannian volume of a compact subset $A \subseteq \mathscr{U}$ as an integral over A.
   (b) Express the length of a differentiable curve $c(t) \in \mathscr{U}$, $t \in [a, b]$, as an ordinary integral over the interval $[a, b]$.
5. Read Example 2.6 and Exercise 7, page 46, of do Carmo's text. We will do more with geometry of Lie groups later.