1. The point $P = (2, 35)$ is on the graph of $f(x) = 3x^2 + \frac{46}{x}$. What is the equation of the line perpendicular to the tangent line to the graph at $P$?

A) $y = -2x + 39$   B) $y = 2x + 31$   C) $y = \frac{1}{2}x + 34$

D) $y = -\frac{1}{2}x + 36$   E) $y = x + 33$   F) $y = -2x + 36$

G) $y = 4x + 27$   H) $y = -\frac{1}{4}x + 37$   I) $y = 5x + 25$

J) $y = 3x + 29$
2. Suppose \( h(x) = \sin(\pi f(x)) \) and that
\[
f(0) = 1 \quad f'(0) = 3
\]
What is \( h'(0) \)?

A) \(-4\pi\)  B) \(-3\pi\)  C) \(-2\pi\)  D) \(\pi\)  E) 0
F) \(\pi\)  G) \(2\pi\)  H) \(3\pi\)  I) \(4\pi\)  J) \(5\pi\)

3. The point \( P = (1, 0) \) is on the graph of \( xe^y + y = 1 \). What is the slope of the tangent line at \( P \)?

A) 0  B) 1  C) 2  D) \(e\)  E) \(e^2\)
F) \(-e\)  G) \(-\frac{1}{2}\)  H) \(-e^2\)  I) \(2e\)  J) \(\frac{1}{4}\)
4. The graph shows two functions $f(x)$ and $g(x)$:

Let $h(x) = f(x)g(x)$. What is $h'(2)$?

A) 5  B) 4  C) 3  D) 2  E) 1  
F) 0  G) −1  H) −2  I) −3  J) −4

5. If $y = g(x) = \log_{10}(x^3 + x^2 + 2x + 1)$, what is $g'(1)$?

A) $\frac{7}{50}$  B) $\frac{1}{5 \ln(10)}$  C) $\frac{7}{5 \ln 10}$  D) $\frac{7}{5}$  E) $\frac{1}{5}$  
F) $\frac{1}{2 \ln(10)}$  G) $\frac{7}{5 \log_{10}(10)}$  H) $\frac{1}{5 \log_{10}(10)}$  I) 0  J) 1
6. A particle is moving along the $x$-axis. Its position at time $t$ is
\[ s = f(t) = \frac{1}{3}t^3 - \frac{5}{2}t^2 + 4t + 1. \] For what times $t$ is the particle moving to the left?

A) $t > 0$  
B) $t > 1$  
C) $0 < t < 2$  
D) $1 < t < 4$

E) $t > 4$  
F) $-1 < t < 0$  
G) $-2 < t$  
H) $-4 < t < -1$

I) $-2 < t < 2$  
J) $-4 < t < 4$

7. What is $\lim_{h \to 0} \frac{\arctan(2 + h) - \arctan(2)}{h}$?

A) 0  
B) 1  
C) 2  
D) 4  
E) $\pi$

F) $\frac{\pi}{4}$  
G) $\frac{\pi}{2}$  
H) $-\frac{1}{2}$  
I) $\frac{1}{5}$  
J) DNE
8. We start with 10 g of a radioactive substance unobtainium. It decays into other isotopes at a rate proportional to the amount the amount of unobtainium present. After 1 minute, only 5g of unobtainium remains. How much unobtainium remains after 3.4 minutes?

A) $2^{-3.4}$ g  
B) $2^{-6.8}$ g  
C) $2^{-1.7}$ g  
D) $5 \cdot 2^{-3.4}$ g  
E) $5 \cdot 2^{-6.8}$ g  
F) $5 \cdot 2^{-1.7}$ g  
G) $10 \cdot 2^{-3.4}$ g  
H) $10 \cdot 2^{-6.8}$ g  
I) $10 \cdot 2^{-1.7}$ g  
J) $\frac{e}{2}$ g

9. What is $\lim_{x \to 0} \frac{\sin^{2}x \sin 6x}{8x^{2}}$?

A) $\frac{1}{4}$  
B) $\frac{3}{4}$  
C) $\frac{3}{2}$  
D) 1  
E) $\frac{5}{2}$  
F) 2  
G) 6  
H) 8  
I) 12  
J) DNE

10. For times $t > 0$ (sec), a particle moving along a straight line has position $s = \frac{3t^{3}-2t^{2}+t}{t}$ (ft). What is its velocity at time $t = 2$?

A) 0 ft/sec  
B) $\frac{1}{4}$ ft/sec  
C) 2 ft/sec  
D) $\frac{10}{4}$ ft/sec  
E) 3 ft/sec  
F) $\frac{35}{4}$ ft/sec  
G) 10 ft/sec  
H) 11 ft/sec  
I) $\frac{23}{2}$ ft/sec  
J) 13 ft/sec
11. If \( f(t) = \ln \left( \frac{t^2}{(t+1)e^t} \right) \), what is \( f'(1) \)?

A) 0 
B) 1 
C) 2 
D) 3 
E) 4 
F) \( \frac{1}{2} \) 
G) \( \frac{1}{3} \) 
H) \( \frac{1}{4} \) 
I) \( \frac{1}{5} \) 
J) \( -1 \)

12. Simplify \( \cos^2(\arctan b) \)

A) \( b^2 \) 
B) \( 1 + b^2 \) 
C) \( 2b^2 \) 
D) \( \frac{1}{1+b^2} \) 
E) \( \frac{b}{1+b^2} \) 
F) \( \frac{b^2}{1+b^2} \) 
G) \( \frac{b}{b^2 - 1} \) 
H) \( \frac{4b}{4+b^2} \) 
I) \( \frac{b}{\sqrt{1+b^2}} \) 
J) \( \frac{2b}{\sqrt{1+2b^2}} \)
13. If \( f(x) = \sin^2 x \), what is \( f'(\frac{\pi}{6}) \) ?

A) \( \frac{\sqrt{3}}{2} \)  
B) \( \frac{\sqrt{2}}{2} \)  
C) \( \frac{1}{2} \)  
D) 1  
E) 0  
F) \( -\frac{\sqrt{3}}{2} \)  
G) \( -\frac{\sqrt{2}}{2} \)  
H) \( -\frac{1}{2} \)  
I) \( -\frac{1}{3} \)  
J) \( -\frac{1}{4} \)

14. The function \( y = f(x) = \ln\left(\frac{4a+x^2}{e^x}\right) \) has a horizontal tangent line at \( x = 1 \). What is the value of \( a \) ?

A) 4  
B) 2  
C) \( \frac{1}{2} \)  
D) \( \frac{1}{4} \)  
E) 0  
F) \( \frac{5}{e} \)  
G) \( \frac{1}{e} \)  
H) \( \frac{4}{e^2} \)  
I) \( -\frac{1}{2} \)  
J) \( -2 \)

Questions 15 - 19 are “true/false” questions (worth 1 point each)

15. \( \frac{d}{dx} \ln(\pi) = \frac{1}{\pi} \)

A) True  
B) False

16. If \( f(x) \) is not differentiable at \( a \), then \( f(x) \) cannot be continuous at \( a \).

A) True  
B) False

17. If \( f'(x) = g'(x) \) for every \( x \), then it must be that \( f(x) = g(x) \) for every \( x \).

A) True  
B) False
18. A cubic polynomial function \( y = ax^3 + bx^2 + cx + d \) must have at least one horizontal tangent line to its graph.

A) True  B) False

19. A point moves along the \( x \)-axis with position \( s = t^3 - 3t^2 \) at time \( t \). For \( 0 < t < 1 \), its speed is increasing.

A) True  B) False
PART II (25 points)

Name ________________________________ WUSTL ID __________

Circle the time of your discussion section on the line that has your TA's name:

Mr. Gong Cheng,  8 a.m.  9 a.m.  10 a.m.  11 a.m.
Mr. Cody Stockdale,  8 a.m.  9 a.m.  10 a.m.  12 p.m.
Ms. Wei Wang,  10 a.m.  11 a.m.  12 p.m.

_________________________________________________________________

For each problem, clearly show your solution in the space provided. “Show your solution” does not simply mean “show your scratch work” — you should cross out any scratch work that turned out to be wrong or irrelevant and, where appropriate, present a readable, orderly sequence of steps showing how you got the answer. A correct answer without supporting work may not receive full credit.
1. Here is the graph of a function $y = f(x)$. The grid lines on both axes in the picture are one unit apart.

a) For $-8 < x < 8$:

List all the $x$'s for which $f'(x)$ does not exist:

For what $x$'s is $f'(x) > 0$?

For what $x$'s is $f'(x) = 0$?

b) On the same grid, draw a reasonable graph for $y = f'(x)$. Use an solid dot • where needed to emphasize that a point is on the graph, and an open dot o to indicate that a point is not included on the graph.
2. Find each derivative. You do not need to simplify after you have finished differentiating, BUT

\[ \text{DRAW A BOX AROUND YOUR FINAL ANSWER IN EACH PART} \]

a) If \( s = f(t) = 2(t^2 - 3t)^3 \), then

\[ \frac{ds}{dt} = \]

b) If \( y = f(x) = \arcsin(3x^2) + \arctan(x^2) \), then

\[ f'(x) = \]

c) If \( y = \tan(\sin x) + 3x^2 \), then

\[ f'(x) = \]

d) If \( y = x^{\cos x} \), then \( \ln(y) = \ln(x^{\cos x}) = \cos x \ln x \).

Therefore \( \frac{y'}{y} = (\cos x) \cdot \frac{1}{x} - (\sin x)\ln x \), so

\[ y' = \]
3. The picture shows the graph of $x^2 + 3xy + 4y^2 = 1$. It is an ellipse that contains the point (1, 0).

\[ y \]

\[ -1.5 \quad -1.0 \quad -0.5 \quad 0.5 \quad 1.0 \quad 1.5 \]

\[ -0.5 \quad 0.5 \]

a) Let $m$ be the slope of the tangent line at (1, 0). Write the equation of the tangent line (in terms of $m$). Put your final answer in the box below.

Tangent line at (1, 0):

b) $m$ is the value of the derivative $y' = \frac{dy}{dx}$ at the point (1, 0). Find the value of $m$. Put your final value for $m$ in the box below.

$x^2 + 3xy + 4y^2 = 1$. Differentiate both sides with respect to $x$ to get:

\[ 2x + 3x'y + 3y + 8yy' = 0. \]

Setting $x = 1$ and $y = 0$ we get

\[ 2 + 3y' + 0 + 0 = 0, \text{ so } y' = -\frac{2}{3}. \]

Final answer: $m =$
DO NOT WRITE ON THIS PAGE. IT IS FOR SCORING PURPOSES ONLY:

Part II Scores

Q1: / 9

Q2: / 8

Q3: / 8

TOTAL PART II: / 25