<u>One way not to write a proof</u>

Theorem If a point x in $(X, \mathcal{T})$ is not isolated, then every open set that contains x is infinite.

Proof Assume not: suppose x is not isolated and that there is a finite open set that contains x.

Since x is not isolated, we can choose distinct points x_n such that $(x_n) \to x$.
(... *details about why omitted for purposes of this example*...)

If O is any open set that contains x, the sequence (x_n) is eventually in O,
so every open set O that contains x is infinite.

This contradicts our assumption that there is a finite open set that contains x.

Therefore the theorem is true. •

The proof (*after filling in a couple of missing details*) is logically correct – but the logic, at best, is unnecessarily confused. Here's an analysis of the logic in the preceding proof, beginning with labels on some of the some parts.

Theorem If <u>x is not isolated</u> (P) in $(X, \mathcal{T})$, then <u>every open set that contains x is infinite</u> (Q).

Proof Assume not: that <u>x is not isolated</u> (P) and <u>there is a finite open set that contains x</u> (∼ Q).

> Since <u>x is not isolated,</u> (P) we can choose distinct points x_n such that $(x_n) \to x$.
> (...*some details about why omitted for this example*...)
>
> If O is any open set that contains x, the sequence (x_n) is eventually in O,
> so <u>every open set O that contains x is infinite</u> (Q).

Since we assumed (∼ Q) that there is a finite open set that contains x, we have contradicted our assumption.

Therefore the theorem is true. •

The proof is presented as a "proof by contradiction." But notice that the argument in the box, by itself, is a complete direct proof of the theorem. The boxed argument has logical form:

Assume	P
Argue that	$P \Rightarrow Q$
Therefore	Q (as desired).

In the long version, the opening assumption $\sim Q$ is never actually used in the rest of the argument. It is there simply as a "straw man" to be contradicted at the end.

The complicated logic of the longer version is:

Assume	P and $\sim Q$.
Argue that	$P \Rightarrow Q$ (*direct proof, not using assumption* $\sim Q$)
Therefore	Q

But this contradicts the assumption $\sim Q$
Since we got a contradition, we conclude Q.

A "genuine" proof by contradiction would assume P and $\sim Q$ and use both assumptions to derive a contradiction of some known previous known result:

For example: Assume P and $\sim Q$.

(*Argument using both of these assumptions*) ..., so $\sqrt{2}$ is rational.

But this is impossible, so our assumption was wrong.