Math 4171-4181
2013-2014
Topology I, II

We can divide mathematics into a few very broad areas such as analysis, algebra, discrete mathematics, topology, geometry, and foundations of mathematics. Some might differ with this breakdown – for example they might add probability theory, while others would just view probability theory as part of analysis (continuous random variables) and part of discrete mathematics (discrete random variables).

Such an artificial breakdown is certainly not important since the boundaries are very fuzzy and all the parts of mathematics interact with each other. But almost everyone would agree that if mathematics is partitioned into its "major parts," then topology would be one large piece. For example, you can find topologists who call themselves general topologists, algebraic topologists, differential topologists, low-dimensional topologists, geometric topologists, piecewise linear topologists, combinatorial topologists, ...

Math 4171 (and Math 4181 in the spring) will be devoted primarily to general topology: that is where topology began historically as a distinct field of mathematics. As the name suggests, general topology contains the broad foundational material for the whole subject. Not only does it contain some beautiful ideas and theorems, but it is also absolutely essential for further work in topology and in other fields like functional analysis. Near the end of the second semester (Math 4181) we might spend a few weeks introducing a little algebraic topology: that will depend partly on the class and on how quickly other topics move along.

As a field, general topology (*once called by a more old-fashioned name,"point-set topology"*) is no longer very research-active – by now it is a well-defined and time-tested body of results. Most of the interesting questions have been settled. Some research does still continue, but most of it is closely related to set theory, mathematical logic and foundations of mathematics: for example, a long-standing classical problem in general topology called the "normal Moore space conjecture" turned out to be unprovable from the standard axioms ZFC for set theory. The important topological research today is in sub-fields such as algebraic topology and differential topology. However, a good foundation in general topology is essential to be able to work in those areas.

Faculty members have different philosophies about our Math 4171-4181 sequence. Some make Math 4181 substantially into a course on algebraic topology. Although that's an exciting topic, I have two problems with this:

- Some knowledge of algebra, especially group theory and some ring theory, is necessary to do algebraic topology: the subject is all about using tools from algebra to study topological questions. But an algebra course like our Math 430 is not a prerequisite for Math 4181, and having it as a prerequisite would make the course less accessible for students who are primarily interested in topology as a tool for analysis and who have no particular interest in algebra. Therefore such a 4181 course needs to spend a lot of time to teaching algebra, often in a hurried way that doesn't do justice to the subject.

- Devoting Math 4181 to algebraic topology also means that material from general topology is either very hurried in Math 4171 or that some beautiful and useful classical results are omitted. My personal feeling is that rushing to get to algebraic topology leads to a future sense of anxiety at places where a good command of general topology is really needed. This makes algebraic topology more mysterious and less enjoyable. It's important to take the time to cover general topology carefully and to allow for time and practice to really "digest" the ideas.

The course syllabus for Math 4171 will contain the details about homework, exams and grading. It will be posted online by the time classes start. For now, you should know that the textbook for the course is one that I have written. It has been photocopied and spiral bound, and is available at Hi/Tec Copy Center (http://www.hiteccopy.com/), located at the corner of Big Bend & Forest Park Parkway, near the northwest corner of campus. The cost will be small compared to a regular textbook: about $16.80 + tax. This is just the cost of production + whatever markup Hi/Tec adds for itself to sell the book: nothing goes to me. There will be a similar book for Math 4181 in the spring.

Just so you can see what the course is about, I've included a Table of Contents for the material I plan to cover during in Math 4171 during the fall semester.

Chapter I Sets

Chapter II Metric and Pseudometric Spaces

Chapter III Topological Spaces

Chapter IV Completeness and Compactness

Chapter V Connected Spaces