HW 6:

You may have already solved the problems below, perhaps differently than my
suggestions below. If so, that’s good!

5b) Let $f^n(x)$ be the results of applying $f$ $n$ times to $x$.

i) if possible, pick an $x_0$ so that $x_0 < x_1 < x_2 < \ldots < x_n < \ldots$ where

$$f^n(x_0) = x_n.$$ 

Use the $x_n$’s to find an $(x, x)$ that is a limit point of the graph of $f$

ii) otherwise $\forall x \exists n$ for which $f^n(x) = f^{n+1}(x)$.

Start with 0. For some $n$, $f^n(0) = f^{n+1}(0) = x_1$. 

Then pick $y > x_1$. For some $n$, $f^n(y) = f^{n+1}(y) = x_2$ 

Continue in this way, to define $x_i$ for all $i$. Then look at the points

$(x_i, x_i)$.

5c) Follow the hint given. Assume that for all $x$, $(x, \omega_1) \notin U$

$f(x) \geq \omega_1$. Prove that for some $x$, $f(x) = \omega_1$

(if not, then $f : [0, \omega_1) \to [0, \omega_1)$ and part b) applies to $f$. Therefore ... )

For this $x$, look at $(x, \omega_1)$