1. The graph of two functions \( y = f(x) \) are given in the top row. On the grid below, make a sketch of the graph of \( y = f'(x) \) in each case.
2. Each limit represents \( \frac{dy}{dx} \bigg|_{x=a} \) for some function \( y = f(x) \) and some number \( a \). What are \( f \) and \( a \)?

i) \( \lim_{h \to 0} \frac{\cos(h + \pi) + 1}{h} \)

ii) \( \lim_{x \to 1} \frac{x^4 + x - 2}{x - 1} \)

**Solution:** There is no "method" for doing these. The idea is just to remember that

\[
f'(a) = \lim_{h \to 0} \frac{f(a + h) - f(a)}{a} = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}
\]

and try to match up patterns.

\[
\begin{align*}
\lim_{h \to 0} \frac{f(a + h) - f(a)}{a} &= \lim_{h \to 0} \frac{\cos(h + \pi) + 1}{h} \quad \text{if we choose } f(x) = \cos x \text{ and } a = \pi, \text{ so} \\
\lim_{h \to 0} \frac{\cos(h + \pi) + 1}{h} &= \cos'(\pi) \quad \text{(whatever the value of that is!) and} \\
\lim_{x \to a} \frac{f(x) - f(a)}{x - a} &= \lim_{x \to 1} \frac{x^4 + x - 2}{x - 1} \quad \text{if we choose } f(x) = x^4 + x \text{ and } a = 1. \text{ Then} \\
\text{the limit gives } f'(1).
\end{align*}
\]
3. A point moves along a straight line. At time $t$ its position is $s = f(t)$. The following figure contains the graphs of $f(t)$, $v(t)$ (the velocity function) and $a(t)$ (the acceleration function). Decide which is which.

![Graphs of position, velocity and acceleration for a point moving on a line](image)

**Solution:**

Call the solid graph $f_1$, the dotted graph $f_2$, and the dashed graph $f_3$.

If the dotted graph $f_2$ were the position, $s$: $s$ would be decreasing from 0 to about $t = 1.5$, so $v$ would be $< 0$ for those times; therefore $v$ would have to be the dashed graph $f_3$; by elimination, this would mean $f_3' = \frac{dv}{dt} = a = $ the solid graph $f_1$ — which is impossible: $v$ would be increasing from about 1.5 to about 3.1 but $a$ would be negative there. So $s = f_2$ is impossible.

If the dashed graph $f_3$ were the position, $s$: $s$ would be increasing from about $t = 0$ to about $t = 3.1$, so $v$ would have to be $> 0$ for those times: neither $f_1$ nor $f_2$ could be $v$. So $s = f_3$ is impossible.

Therefore it must be that $s = $ the solid graph $f_1$. Then $s$ is decreasing from $t = 0$ to about $t = 3.1$ so $v$ must be negative for those times. Therefore $v = f_2$ and finally we conclude $a = f_3$. 
4. Let \( f(x) = x^2 \). It is easy to work out from the definition of derivative that \( f'(x) = 2x \). Let \( \ell \) be the tangent line to the parabola \( y = x^2 \) at the point \((1, 1)\). The line \( \ell \) make an angle \( \phi \) where it crosses the positive \( x \)-axis (\( \phi \) is called the "angle of inclination" for \( \ell \)). Find \( \phi \).

Looking at \( \frac{\text{opposite}}{\text{adjacent}} \) in the shaded triangle, we see that \( \tan \phi = \text{slope of the tangent line at (1, 1)} \). This slope is \( f'(1) = 2 \). Therefore \( \tan \phi = 2 \), and \( 0 < \phi < \frac{\pi}{2} \), so \( \phi = \arctan 2 \). This is the exact answer. A calculator can then be used if you want an approximate value for this: \( \arctan 2 \approx 1.1071 \) (radians) which is about 63.4349°.