

Table of Contents

Chapter I Sets

1. Introduction	1
2. Preliminaries and Notation	2
3. Paradoxes	6
4. Elementary Operation on Sets	7
5. Functions	15
6. More About Functions	19
7. Infinite Sets	27
8. Two “Applications”	36
9. More About Equivalent Sets	38
10. The Cantor-Schroeder-Bernstein Theorem	42
11. More About Subsets	45
12. Cardinal Numbers	47
13. Ordering the Cardinals	48
14. The Arithmetic of Cardinal Numbers	50
15. A Final Digression	58
Exercises	5, 13, 25, 32, 55
Chapter Review	59

Chapter II Pseudometric Spaces

1. Introduction	61
2. Pseudometric Spaces	61
3. The Topology of \mathbb{R}	70
4. Closed Sets and Operators on Sets	76
5. Continuity	85
Exercises	74, 82, 96
Chapter Review	99

Chapter III Topological Spaces

1. Introduction	103
2. Topological Spaces	103
3. Subspaces	109
4. Neighborhoods	110
5. Describing Topologies	113
6. Countability Properties of Spaces	126
7. More About Subspaces	131
8. Continuity	135
9. Sequences	142
10. Subsequences	146
Exercises	123, 129, 148
Chapter Review	151

Chapter IV Completeness and Compactness

1. Introduction	154
2. Complete Pseudometric Spaces	154
3. Subspaces of Complete Spaces	157
4. The Contraction Mapping Theorem	165
5. Completions	175
6. Category	178
7. Complete Metrizability	185
8. Compactness	193
9. Compactness and Completeness	200
10. The Cantor Set	203
Exercises	163, 174, 190, 207
Chapter Review	209

Chapter V Connected Spaces

1. Introduction	213
2. Connectedness	213
3. Path Connectedness and Local Path Connectedness	221
4. Components	225
5. Sierpinski's Theorem	229
Exercises	234
Chapter Review	237

Chapter VI Products and Quotients

1. Introduction	239
2. Infinite Products and the Product Topology	239
3. Productive Properties	251
4. Embedding Spaces in Products	261
5. The Quotient Topology	269
Exercises	248, 260, 266, 279
Chapter Review	281

Chapter VII Separation Axioms

1. Introduction	283
2. The Basic Ideas	283
3. Complete Regularity and Tychonoff Spaces	292
4. Normal and T_4 -Spaces	302
5. Urysohn's Lemma and Tietze's Extension Theorem	304
6. Some Metrization Results	314
Exercises	290, 300, 316
Chapter Review	318

Chapter VIII Ordered Sets, Ordinals and Transfinite Methods

1. Introduction	319
2. Partially Ordered Sets	319
3. Chains	323
4. Order Types	331
5. Well-Ordered Sets and Ordinal Numbers	337
6. Indexing the Cardinals	350
7. Spaces of Ordinals	352
8. The Spaces $[0, \omega_1)$ and $[0, \omega_1]$	354
9. Transfinite Induction and Recursion	361
10. Using Transfinite Induction and Recursion	363
11. Zorn's Lemma	374
Appendix: Exponentiation of Ordinals	384
Exercises	330, 335, 359, 381
Chapter Review	386

Chapter IX Convergence

1. Introduction	388
2. Nets	388
3. Filters	394
4. The Relationship Between Nets and Filters	397
5. Ultrafilters and Universal Nets	401
6. Compactness Revisited and the Tychonoff Product Theorem	405
7. Applications of the Tychonoff Theorem	408
Exercises	411
Chapter Review	416

Chapter X Compactifications

1. Basic Definitions and Examples	418
2. Local Compactness	420
3. The Size of Compactifications	423
4. Comparing Compactifications	425
5. The Stone-Cech Compactification	431
6. The Space $\beta\mathbb{N}$	438
7. Alternate Constructions of βX	443
Exercises	447
Chapter Review	449