Sequences and Series on the TI-83

There are a number of useful things you can do with the TI-83 in Chapter 8.

To work with sequences, first change the graphing mode: Hit the MODE key, and use the arrow keys to move down and over to highlight the SEQ mode. (*For some of the examples below, this isn't necessary, but it's safer just to do it at the beginning.*) Then use 2nd QUIT to return to the main screen.

1. Looking at the terms of a sequence:

Consider a sequence $\{a_n\}_{n=1}^{\infty}$. For example, say $a_n = \frac{\sin n}{n}$. The sequence starts out as $\{ \frac{\sin 1}{1}, \frac{\sin 2}{2}, \frac{\sin 3}{3}, \ldots \} \simeq \{.84147, .45465, .04704, \ldots\}$

You can list some of the terms of this sequence using the *seq* command on the TI-83. It is found using the sequence of menus: 2nd LIST, OPS. The *seq* command is number 5 on the list. To use the command, the syntax is

seq(*formula for* a_n, n, a, b)

The command produces a list of the values of a_n, starting with $n = a$ and ending with $n = b$.

For example, the command

seq(sin(n)/n, n, 1, 50)

generates the first 50 terms of the sequence $a_n = \frac{\sin n}{n}$. (The list runs off the screen to the right; use the left/right arrow keys to scroll through the list.)

Note: in the seq command, the letter "n" is irrelevant; use any letter you like. However, since you're in the SEQ mode, hitting the key labeled X, T, θ, n *will automatically enter the letter "n" for you.*

2. Making a table of sequence values

Hit the Y= key. Since you're in the *SEQ* mode, the Y= window is set up for entering sequences. You can enter up to 3 sequences, using the names $u(n)$, $v(n)$ or $w(n)$ for the formula for a_n. (These are analogous to Y1= , Y2= , Y3= when you're in the function graphing mode.)

Let's say you're working with just one sequence, with $a_n = \frac{\sin n}{n}$. If you want to start with $n = 1$, enter nMin = 1, and, on the second line, fill in the blank after $u(n)$, so it reads $u(n) =$ sin(n)/n. (*For now, the third line, u(nMin) can be left blank*).

Then hit the 2nd TABLE key. The values of n and $u(n) = a_n = \frac{\sin n}{n}$ are displayed. Use the up/down arrow keys to scroll through the table.

(*If you like, you can vary the table using the options under 2ns TBLSET: TblStart changes the starting n value at the top of the table when the table first appears on screen;* ΔTbl *determines the step size – if TblStart=1 and* $\Delta Tbl = 2$, *the table will display values of the sequence only for* $n = 1, 3, 5, \ldots$
If Indept is set to Ask, you have to enter the n values you're interested in by hand; if Depend is set to Ask, you have to highlight the $u(n) = a_n$ *value you want and hit enter to compute that value.*)

3. Graphing a sequence (plotting the points (n, a_n) in the plane)

Once you've entered the value for nMin and the formula for $u(n) = a_n$ in the Y= screen, the GRAPH key will plot the points (n, a_n). Of course, you may have to use the WINDOW key to reshape the window to "fit" the part of the sequence you want to see.

Since you're in the SEQ mode, the WINDOW screen looks a bit different. You use xMin, xMax, yMin, yMax just as when you graphed functions. nMin and nMax give the largest and smallest values of n to be used. *(nMin will automatically be set equal to whatever you entered in the Y= screen; changing nMin in either place automatically changes it in the other.)* PlotStart gives the value of n for the first point to be plotted (perhaps not the same as the smallest n used in the "background" calculations). PlotStep gives a stepsize. *(For example, if PlotStart = 2 and PlotStep = 3, the points (n, a_n) will be plotted only for n=2, 5, 8, ... Usually you'll set PlotStart the same as nMin and PlotStep = 1.)*

You can set xMin=nMin and xMax = the largest value of n you want to see plotted. You can estimate the necessary sizes for Ymin and Ymax by looking at a table of values (see 2. above)

Example: Enter $u(n) = a_n = \frac{3n+7(-1)^n}{4n+5}$, and set nMin=1, nMax=100, Xmin=1, Xmax=100, Ymin= – 1, Ymax = 2 and then GRAPH. You can see plotted the first 100 points (n, a_n). They appear to approach a horizontal asymptote somewhere around 0.75 *(Can you show, using limits, that the exact value of the limit is $\frac{3}{4}$?)*

Once the graph is on the screen, you can hit the TRACE key to move along the plotted points and read off the values of n and a_n at each point.

4. Making a Table or Graphing a recursively defined sequence

A recursively defined sequence is one where you are not given a formula for a_n in terms of n (such as $a_n = \frac{3n+7(-1)^n}{4n+5}$), but instead are given a starting value and a formula for computing the rest of the a_n's in terms of the previous ones: for example, $a_1 = 2$ and $a_n = \frac{1}{2}(a_{n-1} + 6)$. This specific sequence is Example 11 on pp. 566-567 of the textbook.

To make a table or plot this sequence, the steps are the same as above except for how you enter the sequence in the Y= window. For example, for the sequence above, you'd enter

nMin =	1
u(n) =	.5*(u(n – 1)+6) *(NOTE: to enter the term u(n – 1) in the right side, you MUST use the special "u" key for sequences on the keyboard: NOT ALPHA U, but 2nd 7 (the u above the 7 key). Similarly, you'd have to use the keys 2nd 8 or 2nd 9 to refer to the sequence v(n) or w(n) if you were using those lines.)*
u(nMin)=	2 *(This line you ignored before. For a recursively defined sequence, you have to provide an "initial value". You've already set nMin = 1, so here you're saying that u(nMin) = u(1) = 2, that is, that a_1 = 2, as given in the example.)*

Now, the TABLE key gives the consecutive values of $u(n) = a_n$ for each value of n. The a_n's seem to be approaching 6. If you set the window with nMin=xMin=1, nMax=xMax=30 (say), yMin=0, yMax=7 (for a little extra room), you can see the plotted points leveling out toward an asymptote at height somewhere around 6.

In the textbook, the example shows that 6 is actually the exact value of the limit.

5. Working with infinite series

When we consider whether or not an infinite series such as $\sum_{n=1}^{\infty} a_n$ converges, we mean that we look at a special sequence which we build from it, the sequence of partial sums, in which we "add up a bit more and a bit more" and see if we approach a limit. That is, we look at

$$
\begin{aligned}
s_1 &= a_1 && \text{(add up terms out to } n = 1\text{)} \\
s_2 &= a_1 + a_2 && \text{(add up terms out to } n = 2\text{)} \\
s_3 &= a_1 + a_2 + a_3 && \text{(add up terms out to } n = 3\text{)} \\
&\dots \\
s_n &= a_1 + a_2 + a_3 + \dots + a_n && \text{(add up terms out to } n\text{)}
\end{aligned}
$$

and see whether the sqeuence of partial sums (as the running total "accumulates") approach a limit. If $\lim_{n \to \infty} s_n = L$, we say the series converges and write that the infinite sum $\sum_{n=1}^{\infty} a_n = L$.

Example: Consider the series $\sum_{n=1}^{\infty} \frac{1}{2^n}$. The partial sums are

$s_1 = \frac{1}{2}$, $\quad s_2 = \frac{1}{2} + \frac{1}{2^2}$, $\quad s_3 = \frac{1}{2} + \frac{1}{2^2} + \frac{1}{2^3}$, and in general

$$s_n = \frac{1}{2} + \frac{1}{2^2} + \frac{1}{2^3} + \dots + \frac{1}{2^n} = \sum_{i=1}^{n} \frac{1}{2^i}.$$

We can look at this sequence on the TI-83, using the SUM function that we used when doing Riemann sums: it's on the menu 2nd LIST MATH (item #5)

Recall that the SUM function adds up the numbers in a list. To compute s_n, what we want is to add up the numbers in the list $\frac{1}{2}, \frac{1}{2^2}, \frac{1}{2^3}, \dots \frac{1}{2^n}$. And, as in part 1 above, we use the SEQ function (found at 2ns LIST OPS, item #5) to create the list. The list is created with the command

seq(1/2^I, I, 1, n)

Therefore, we go to the Y= window and enter the formula for s_n opposite $u(n)$:

nMin = 1
$u(n)$ = sum(seq(1/2^I, I, 1, *n*)) *(the formula for s_n)*
(the line $u(nMin)$= should be left empty; clear it if necessary)

Now, just as above, we can use the TABLE and GRAPH commands (after adjusting the WINDOW, if necessary) to create a table of the values of s_n, or a graph of the sequence s_n.

In this example, the TABLE command produces a table that looks like:

n	$u(n)$ (that is, s_n)	
1	.5	$(= \frac{1}{2})$
2	.75	$(= \frac{1}{2} + \frac{1}{2^2})$
3	.875	$(= \frac{1}{2} + \frac{1}{2^2} + \frac{1}{2^3})$
4	.9375	
5	.96875	
	(skipping down)	
10	.99902	
	(skipping some more)	
17	.99999	

(For large n, each calculation of $u(n) = s_n$ *takes a while, since ther are a lot of terms to add up!)*

It looks as if the s_n's approach a limit = 1 as $n \to \infty$. So we guess, from this data, that $\sum_{n=1}^{\infty} \frac{1}{2^n} = 1$.

This is also suggest by the graph: set the window with nMin = xMin = 1, nMax = xMax = 17 (say), yMin=0, yMax=2. The plotted terms of the sequence s_n (of partial sums for the series) level off toward an asymptote that looks to be at height about 1. In fact, TRACE along the points and watch the 2nd coordinates; they're the same as the s_n values in the table above.

In fact, as we saw in class, this is a geometric series with first term $a = \frac{1}{2}$ and ratio $r = \frac{1}{2}$, so we actually know that it converges (because $|\, r \,| < 1$) and its sum is exactly

$\frac{a}{1-r} = \frac{\frac{1}{2}}{1-\frac{1}{2}} = 1$, just as the calculator suggests.