Math 5061: Theory of Statistics I

Fall 2014

Professor: Todd Kuffner

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Lectures:	Mon and Wed 8:30-10am, Room 199, Cupples I
Office Hours:	Tues 9:30-10:30am, Wed 10-11am or by appointment

Course Description: This course is intended for mathematics and statistics Ph.D. students. Math 5061 is the first half of a year-long course which forms the basis for the Ph.D. qualifying exam in theoretical statistics. Math 5061-5062 will provide graduate students with a rigorous introduction to probability theory and an overview of both finite sample theory and large sample theory in statistics. A list of topics is given below.

Prerequisites: At a minimum: solid preparation in probability and statistics at an advanced undergraduate level, as well as undergraduate-level analysis.

Course Format: It is better to regard Math 5061-5062 as two distinct courses: Math 5061p-5062p (probability) and Math 5061s-5062s (statistics). The course will begin with several weeks of measure theory needed for both the probability and statistics material. Thereafter, lectures will be divided between probability and statistics. It will often be necessary to introduce concepts in Math 5061s which will not be treated rigorously until they are studied later in Math 5061p or Math 5062p (in the second semester); this is why it is strongly recommended that students already have some familiarity with probability and statistics. Naturally, we will need to proceed rapidly, and we will not be able to cover all topics at the ideal depth. For this reason, students are strongly encouraged to concurrently enroll in Math 5051-5052, which is the graduate sequence in measure theory and real analysis.

Reference Book for Math 5061p and 5062p:

- Athreya, K.B. and S.N. Lahiri. *Measure Theory and Probability Theory, 1st edition*, Springer.

Reference Books for Math 5061s and 5062s:

- Lehmann, E.L. and G. Casella. *Theory of Point Estimation, 2nd edition*, Springer.
- Lehmann, E.L. and J.P. Romano. *Testing Statistical Hypotheses, 3rd edition*, Springer.

These books are required; readings and problems may be assigned from all three books. The third book is available for free online from the publisher's website (when logging in at WashU).

Homework: You are allowed and encouraged to discuss homework problems with other students. However, the homework you submit must be your own work. Your solutions should be written up neatly; you are encouraged, but not required, to type them in LaTeX.

Grades: Final grades will be determined according to the following weights:

Homework	35%	
Midterm Exam	30%	Wednesday, 15th October
Final Exam	35%	8-10am, Thursday 11th December

Topics List for Math 5061p and 5062p: (*time permitting*)

1. Measures: introduction, extension theorems, Lebesgue-Stieltjes, completeness
2. Integration: measurable transformations, Fubini's theorem, Riemann and Lebesgue integration
3. General topology: L^p-spaces, Banach and Hilbert spaces
4. Differentiation: Radon-Nikodym theorem, signed measures
5. Product Measures, Convolutions and Transforms: Laplace (generating functions), Fourier transforms, Plancherel
6. Probability Spaces: random variables and random vectors
7. Independence: Borel-Cantelli lemmas
8. Laws of Large Numbers: WLLN, SLLN, ergodic theorems

9. Distributions and Quantile Functions
10. Convergence in Distribution: Helly-Bray theorems, Skorohod's theorem, Stein's method
11. Characteristic Functions: Levy-Cramer continuity theorem
12. Central Limit Theorems: Lindeberg-Feller, Berry-Esseen theorem, Edgeworth expansion, large deviations
13. Martingales: discrete parameter martingales, optional sampling algorithm, convergence
14. Markov Chains and MCMC: countable state spaces, general state spaces, Metropolis-Hastings, Gibbs sampler
15. Haar measure: use in Bayesian statistics

Topics List for Math 5061s and 5062s: (*time permitting*)

1. Finite Sample Theory of Point Estimation: statistical models: applications to exponential families, group families, nonparametric families; minimum risk unbiased (and equivariant) estimation, Cramer-Rao inequality
2. Decision Theory: loss functions, risk functions, Bayes estimation, minimax estimation, admissibility, shrinkage estimators
3. Principles of Data Reduction: sufficiency, ancillarity, completeness
4. Finite Sample Theory of Hypothesis Testing and Confidence Intervals: Neyman-Pearson theory, UMP/UMPU/UMPI tests, uniformly most accurate confidence intervals, use of conditioning to eliminate nuisance parameters in exponential families; use of invariance to eliminate nuisance parameters in group families; maximin tests and Hunt-Stein theorem, permutation tests
5. Large Sample Theory of Point Estimation: asymptotic relative efficiency, maximum likelihood estimation, chi-square tests, Rao, Wald and likelihood ratio tests, delta method, asymptotic distribution of quantiles and trimmed means, differentiability of statistical functionals, robustness and influence; rank, permutation and randomization tests; jackknife, bootstrap

6. Asymptotic Optimality Theory of Estimators, Tests and Confidence Intervals: contiguity, quadratic mean differentiability, expansions of the log likelihood ratio, asymptotic minimax theorem, convolution theorem, asymptotically uniformly most powerful tests