(1) Prove that for sets $A, B, C$, $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$.

(2) If $f : A \to B$ and $g : B \to C$ are functions and $g \circ f : A \to C$ is injective, prove that $f$ is injective.

(3) Let $f : A \to A$ be a function such that $(f \circ f)(a) = a$ for all $a \in A$. Prove that $f$ is bijective.

(4) For any $n \in \mathbb{N}$, let $\Sigma_n = \{m \in \mathbb{N} | m \leq n\}$. If $f : \Sigma_{n+1} \to \Sigma_n$ is any function, prove that $f$ is not injective. (This is usually called the ‘Pigeon Hole principle’.)

(5) If $x \geq 0$ is a real number and $n \in \mathbb{N}$, prove that $(1+x)^n \geq 1+nx$.

(6) Let $n \in \mathbb{N}$. Prove that there exists a rational number $x$ with $x^2 = n$ if and only if there is a natural number $m$ with $m^2 = n$.

(7) Prove that given any prime number, there is a larger prime.

(8) Let $a \in \mathbb{Z}$ with $p$ not dividing $a$ for a prime $p$. Prove that there is a natural number $n$ such that $p|a^n - 1$. (Hint: Look at remainders of $a^n - 1$ when divided by $p$ and use the Pigeon Hole principle.)