Math 310, Homework 5, due 8th October 2012

You must justify all your answers, by giving a proof of your answer. You may use
any property of numbers you have studied.

(1) Assume we define $\Gamma \subset \mathbb{R} \times \mathbb{R}$ as below. Decide whether it is the graph of a
function from $\mathbb{R}$ to $\mathbb{R}$.
   (a) $\Gamma = \{(x, y)|x > y\}$.
   (b) $\Gamma = \{(x, y)|x = y\}$.
   (c) $\Gamma = \{(x, y)|x^2 = y\}$.
   (d) $\Gamma = \{(x, y)|x = y^2\}$.
   (e) $\Gamma = \{(x, y)|x^2 = y^2\}$.

(2) Decide whether the functions below are injective or surjective.
   (a) $f : \mathbb{R} \to \mathbb{R}$, given by $f(x) = \sin x$.
   (b) $f : \mathbb{R} \to \mathbb{Z}$, given by $f(x) = \lfloor x \rfloor$ where the right hand side is the largest
   integer $a \in \mathbb{Z}$ with $a \leq x$. (For example, $f(5) = 5, f(\pi) = 3, f(e) = 2$
   etc.)
   (c) $f : \mathbb{R} \to \mathbb{R}$, given by $f(x) = |x|$.
   (d) $f : \mathbb{R} \to \mathbb{R}$, given by $f(x) = e^x$.
   (e) $f : \mathbb{R} \to \mathbb{R}$, given by $f(x) = \int_0^x e^t \, dt$.