HOMEWORK 11

DUE NOVEMBER 30

(1) If $\{x_n\}$ is a CS, show that so is $\{|x_n|\}$. (The sequence could be in $\mathbb{Q}$ and so do not use limit arguments etc. The same applies to the next problem.)

(2) Show that the sequence $\{x_n\}$, where $x_n = \sum_{k=1}^{n} a^k$ is a CS if and only if $|a| < 1$.

(3) Let $I_n = [a_n, b_n]$ be non-empty closed intervals in $\mathbb{R}$ for all $n \in \mathbb{N}$. (Non-empty just means $a_n \leq b_n$.) Further assume that $I_{n+1} \subset I_n$ for all n. (Again, this just means, $a_n \leq a_{n+1} \leq b_{n+1} \leq b_n$). Show that $\cap_{n=1}^{\infty} I_n \neq \emptyset$. (Hint: Do not forget lub).

(4) Let $S \subset I = [a, b]$ be an infinite subset. Show that there exists an injective map $f : \mathbb{N} \to S$ such that the corresponding sequence is a CS.