HOMEWORK 12

DO NOT SUBMIT

(1) Show that if U_α with $\alpha \in \Lambda$ is an indexed collection of open sets of $\mathbb{R}$, $\cup_{\alpha\in\Lambda} U_\alpha$ is open. Show also that intersection of finitely many open sets is open, but intersection of infinitely many open sets may not be open. (Statements about open sets will naturally have a dual statement for closed sets).

(2) Show that the open interval (a, b) $(a < b)$ is open and the closed interval $[a, b]$ is closed.

(3) Show that a subset $X \subset \mathbb{R}$ is closed if and only if for any CS $\{x_n\}$ with $x_n \in X$, $\lim x_n \in X$.

(4) Show that if $U \subset \mathbb{R}$ is both open and closed, then $U = \emptyset$ or $U = \mathbb{R}$. (This is usually referred to as $\mathbb{R}$ is *connected*).

(5) Let $f : \mathbb{R} \to \mathbb{R}$ be a continuous function and assume that $f(x) \neq 0$ for all $x \in \mathbb{R}$. Show that $\frac{1}{f}$ is a continuous function from $\mathbb{R}$ to $\mathbb{R}$.

(6) If $f, g : \mathbb{R} \to \mathbb{R}$ are continuous functions, show that so is $g \circ f$.