HOMEWORK 2

DUE SEPTEMBER 19

As usual, you can only use the basic properties of integers stated in class, including any integer is either even or odd (and not both) and definitions of odd and even for the following problems. Similar properties for real numbers also may be assumed.

(1) Let P, Q, R be statements. Prove the logical equivalence of two statements in each of the problems below by writing down a truth table.
   (a) $\neg(P \wedge Q)$ and $(\neg P) \vee (\neg Q)$.
   (b) $P \Rightarrow Q$ and $(\neg P) \vee Q$.
   (c) $P \wedge (Q \vee R)$ and $(P \wedge Q) \vee (P \wedge R)$.
   (d) $P \vee (Q \wedge R)$ and $(P \vee Q) \wedge (P \vee R)$. The last two are sometimes called Distributivity.

(2) Using cases, show that if x is an integer then $x(x+1)$ is even.

(3) Write the contrapositive of the statement 'If $x + y$ is odd, then x or y is odd' (x, y are integers). Prove this contrapositive.

(4) Let a, b, c be real numbers.
   (a) Then calculate the number of solutions $ax + b = 0$ where x is a real number.
   (b) Calculate the number of solutions of $ax^2 + bx + c = 0$ where x is a real number. You may assume quadratic formula. (Hint: Separate into cases for these two problems.)