HOMEWORK 3

DUE SEPTEMBER 26

(1) Write the following statements in plain English. (This means, you must not use the symbols, $\neg, \vee, \wedge, \Rightarrow, \forall, \exists, \in$.) Remember that there are many ways of writing in English, but choose the one you think reads best.
  (a) $\forall a \in \mathbb{N}, \forall b \in \mathbb{N}, ab \in \mathbb{N}$.
  (b) $\forall a \in \mathbb{Z}, \exists b \in \mathbb{Z}, a + b = 0$.
  (c) $\exists a \in \mathbb{Z}, \forall b \in \mathbb{Z}, ab = 1$.
(2) Write symbolically the negations of the above statements, without using $\neg$.
(3) Write $S(n)$ for $n \in \mathbb{N}$, the sum $S(n) = \sum_{k=1}^{n}(2k-1)$. Experiment with small values of n and make a guess for $S(n)$ to get a nice formula. Then prove it using induction.
(4) Recall that for any $n \in \mathbb{N}$, the notation $n!$ (pronounced *n factorial*) means $1 \cdot 2 \cdot 3 \cdots (n-1) \cdot n$. Prove by induction, $n! \geq 2^{n-1}$ for all $n \in \mathbb{N}$. Use this to show that $\sum_{k=1}^{n} \frac{1}{k!} \leq 2$ for any $n \in \mathbb{N}$.
(5) If $x > 0$ is a real number and $n \in \mathbb{N}$, prove by induction, $(1+x)^n \geq 1 + nx$. For what values of n can we have strict inequality? (This means, we want $(1+x)^n > 1 + nx$.)