HOMEWORK 4

DUE OCTOBER 3

You may use any result that we proved in class and the ones I have posted for properties of numbers. If you can, quote them by name or statement or by some other identifying feature.

(1) Show that any integer m is of the form $m = 3k$, $m = 3k + 1$ or $m = 3k - 1$, for some $k \in \mathbb{Z}$.

(2) Let m, n be two non-zero integers and p, q be two integers. Assume $\gcd(m, n) = 1$. Then show that there exists an integer N such that $m \mid (N - p)$ and $n \mid (N - q)$.

(3) If $x, y, z \in \mathbb{Z}$ such that $x^2 + y^2 = z^2$, show that either x or y is even.

(4) We discuss an important function called *choose function*. Recall from previous homework, the notation $n!$ for $n \in \mathbb{N}$ stands for $1 \times 2 \times \cdots \times (n-1) \times n$. By convention, we write $0! = 1$. Given $0 \leq r \leq n$, define the choose function $\binom{n}{r}$ (pronounced n choose r) to be,

$$\binom{n}{r} = \frac{n!}{r!(n-r)!}.$$

(a) If $n \geq 2$ and $1 \leq r < n$, show that,

$$\binom{n}{r} = \binom{n-1}{r} + \binom{n-1}{r-1}.$$

(b) Use the above to show that for any $n \in \mathbb{N}$ and $0 \leq r \leq n$, $\binom{n}{r} \in \mathbb{N}$.

(5) Prove the binomial theorem: if $x, y \in \mathbb{R}$ and $n \in \mathbb{N}$, then,

$$(x + y)^n = \sum_{r=0}^{n} \binom{n}{r} x^r y^{n-r}.$$

(6) Imitate the proof we gave in class to show that there is no rational number r such that $r^3 = 2$.