# HOMEWORK 6

DUE OCTOBER 19

*You may use any result that we proved in class. If you can, quote them by name or statement or by some other identifying feature.*

(1) Decide whether the following are true. If true, give a proof and if false, give a counterexample to illustrate your reasoning.
   (a) If $A, B$ are subsets of $\Omega$ and $A \cap B = \emptyset$, then $A = \emptyset$ or $B = \emptyset$.
   (b) If $A$ is not contained in $B$ and $C \subset B$, then $A$ is not contained in $C$.

(2) We generalize the notions introduced in class for possibly infinite collections as follows. Let $\Lambda$ be a non-empty set, possibly infinite. We denote by Greek letters, elements of $\Lambda$.

   Assume $\Omega$ is a fixed set and we are given subsets $A_\alpha \subset \Omega$ for each $\alpha \in \Lambda$. (We have *indexed* the collection of subsets by $\Lambda$). As usual define,

$$\cup_{\alpha\in\Lambda} A_\alpha = \{a \in \Omega | a \in A_\alpha \text{ for some } \alpha \in \Lambda\},$$
$$\cap_{\alpha\in\Lambda} A_\alpha = \{a \in \Omega | a \in A_\alpha \text{ for all } \alpha \in \Lambda\}.$$

   (a) Show that if $B \subset \Omega$, then $B \cap (\cup_{\alpha\in\Lambda} A_\alpha) = \cup_{\alpha\in\Lambda}(B \cap A_\alpha)$.
   (b) Show that $(\cup_{\alpha\in\Lambda} A_\alpha)^c = \cap_{\alpha\in\Lambda} A_\alpha^c$.
   (c) For this problem, we use $\Lambda = \mathbb{N}$ (so instead of Greek letters, we use English letters) and $\Omega$ to be the open interval $(0, 1)$. Write down a collection of subsets $A_n, n \in \mathbb{N}$ of $(0, 1)$ such that for any $j, k \in \mathbb{N}$, $A_j \cap A_k \neq \emptyset$, but $\cap_{n\in\mathbb{N}} A_n = \emptyset$. (Hint: A real number $x$ is positive is the same as saying $x \geq \frac{1}{n}$ for some $n \in \mathbb{N}$.)

(3) Define for two subsets $A, B$ of $\Omega$,

$$A - B = \{a \in \Omega | a \in A, \text{ but } a \notin B\}.$$

   Show that for subsets $A, B, C$ of $\Omega$, $(A - B) - C = A - (B \cup C)$.

(4) Define for two sets $A, B$ as above,

$$A + B = (A - B) \cup (B - A).$$

   Show that $A + A = \emptyset$ and $A + \emptyset = A$ for any $A \subset \Omega$.

(5) If $X, Y \subset A$ and $B$ any other set, show that $(X \cap Y) \times B = (X \times B) \cap (Y \times B)$, all considered as subsets of $A \times B$.