These are some problems about infinite sets.

(1) Let \( A \) be an infinite set and let \( B \subset A \) a finite set. Show that \( A \approx (A - B) \).

(2) If \( A_n, n \in \mathbb{N} \) is a countable collection of countable sets, show that \( \bigcup_{n \in \mathbb{N}} A_n = X \) is countable.

(3) Let \( \mathbb{Z}[X] \) denote the set of all polynomials in \( X \) with integer coefficients. Let \( A_d \subset \mathbb{Z}[X] \) be the subset of all polynomials of degree at most \( d \). (This means a polynomial \( a_0 + a_1X + \cdots + a_nX^n \in A_d, a_n \neq 0 \) if and only if \( n \leq d \).) Show that \( A_d \approx \mathbb{Z} \times \mathbb{Z} \times \cdots \times \mathbb{Z}, \ d + 1 \) copies. Deduce that \( \mathbb{Z}[X] \) is countable.

(4) Show that \( [a, b] \approx [c, d] \), closed intervals with \( a < b, c < d \) of real numbers.

(5) Show that \( [a, b] \approx (a, b), \ a < b \).

(6) Show that \( \mathbb{R} \approx (a, b), \ a < b \).

(7) In class we identified \( F(\mathbb{N}, \{0, 1\}) = X \) with \( [0, \frac{1}{2}] \), by sending \( f \in F(\mathbb{N}, \{0, 1\}) \) to \( \sum_{n=1}^{\infty} f(n)10^{-n} \). Consider the map \( X \times X \) to \( \mathbb{R} \), defined by \( (f, g) \) maps to \( \sum_{n=1}^{\infty} a_n10^{-n} \), where \( a_{2n-1} = f(n), a_{2n} = g(n) \). Show that this map is injective and thus show that \( \mathbb{R} \times \mathbb{R} \approx \mathbb{R} \).