HOMEWORK 9

DUE NOVEMBER 14

These are some problems about infinite sets.

(1) Let A be an infinite set and let $B \subset A$ a finite set. Show that $A \approx (A - B)$.
(2) If $A_n, n \in \mathbb{N}$ is a countable collection of countable sets, show that $\cup_{n \in \mathbb{N}} A_n = X$ is countable.
(3) Let $\mathbb{Z}[X]$ denote the set of all polynomials in X with integer coefficients. Let $A_d \subset \mathbb{Z}[X]$ be the subset of all polynomials of degree at most d. (This means a polynomial $a_0 + a_1 X + \cdots + a_n X^n \in A_d, a_n \neq 0$ if and only if $n \leq d$.) Show that $A_d \approx \mathbb{Z} \times \mathbb{Z} \times \cdots \times \mathbb{Z}$, $d+1$ copies. Deduce that $\mathbb{Z}[X]$ is countable.
(4) Show that $[a,b] \approx [c,d]$, closed intervals with $a < b, c < d$ of real numbers.
(5) Show that $[a,b] \approx (a,b)$, $a < b$.
(6) Show that $\mathbb{R} \approx (a,b)$, $a < b$.
(7) In class we identified $F(\mathbb{N}, \{0,1\}) = X$ with $[0, \frac{1}{9}]$, by sending $f \in F(\mathbb{N}, \{0,1\})$ to $\sum_{n=1}^{\infty} f(n) 10^{-n}$. Consider the map $X \times X$ to $\mathbb{R}$, defined by (f,g) maps to $\sum_{n=1}^{\infty} a_n 10^{-n}$, where $a_{2n-1} = f(n), a_{2n} = g(n)$. Show that this map is injective and thus show that $\mathbb{R} \times \mathbb{R} \approx \mathbb{R}$.