

HOMEWORK 2, DUE SEP 18, 2017

- (1) Write the following in good English without the symbols \forall and \exists .
 - (a) $(\forall x, y \in \mathbb{Z})(x + y \in \mathbb{Z})$.
 - (b) $(\forall x \in \mathbb{Z})(\exists y \in \mathbb{Z})(x + y = 0)$.
 - (c) $(\exists x \in \mathbb{Z})(\forall y \in \mathbb{Z})(xy = 0)$.
- (2) Write the following English sentences symbolically, using \forall, \exists .
 - (a) Square of any real number is non-negative.
 - (b) Given any non-zero real number x , there is a real number y such that $xy = 1$.
 - (c) Given any positive real number ϵ and any real number x , there is a rational number q such that $|x - q| < \epsilon$.
- (3) Write the negations of the following symbolically.
 - (a) $(\forall x \in \mathbb{R})(x^2 > x)$.
 - (b) $(\forall x \in \mathbb{R})(\exists y \in \mathbb{R})(y^2 + y = x)$.
 - (c) $(\forall x, y, z \in \mathbb{Z}, \text{non-zero})(((x \text{ divides } y) \wedge (y \text{ divides } z)) \Rightarrow (x \text{ divides } z))$.
- (4) Using your knowledge of numbers (you may use any thing you know, but state explicitly what you are using if possible), decide which of the above statements (you may skip 2 (c)) are true.
- (5) Describe the following sets using the roster method (i. e. by listing all the elements).
 - (a) $S = \{x \in \mathbb{R} | x^2 + 2x + 1 = 0\}$.
 - (b) $S = \{n \in \mathbb{N} | kn = 10 \text{ for some } k \in \mathbb{N}\}$.
 - (c) $S = \{n \in \mathbb{N} | n^2 < 20\}$.
- (6) Let $A = \{x \in \mathbb{R} | \sin 2x = 0\}$ and $B = \{x \in \mathbb{R} | \cos x = 1\}$. Is $A \subset B$? Is $B \subset A$?