HOMEWORK 3, DUE SEP 25, 2017

For this homework, you can use any property appearing on pages 1-2 of what is in the document on my web page about properties of numbers, except (11), Induction. You may also use anything proved in class and division algorithm.

(1) Let $a, b, c \in \mathbb{Z}$.
   (a) If $a^2 + b^2 = c^2$, show that either a or b is even.
   (b) Find all $m \in \mathbb{Z}$ such that $m^2 + (m+1)^2 = (m+2)^2$.

(2) Show that any integer a can be written as $3m, 3m-1$ or $3m+1$ for some integer m.

(3) Decide which of the following (mathematical) statements are true. You must prove it if you decide it is true or give a counterexample if you decide it is false.
   (a) If $n \in \mathbb{Z}$, $n(n+1)$ is even.
   (b) If $a, b \in \mathbb{Z}$ and ab is even, both a and b are even.
   (c) If $a, b \in \mathbb{Z}$, $|a+b| \leq |a| + |b|$.
   (d) If $a^2 + b^2 = c^2$, at least one of a, b, c is a multiple of 3.

(4) Show that if $2 \mid m$ and $3 \mid m$, then $6 \mid m$, where $m \in \mathbb{Z}$.