(1) In this problem, we show the equivalence of the two induction procedures. So, let \( N \in \mathbb{Z} \) be a fixed integer and let \( A = \{ n \in \mathbb{Z} | n \geq N \} \).

(a) If \( S \) is a non-empty subset of \( A \), it has a minimal element.
(b) Let \( P(n) \) for \( n \in A \) be any open statement. Assume \( P(N) \)
is true and \( P(n) \Rightarrow P(n+1) \) for any \( n \in A \). Then, \( P(n) \)
is true for all \( n \in A \).

Show that the above two versions are equivalent by the following steps.

(a) To show (a) implies (b), consider the set \( S = \{ n \in A | \neg P(n) \} \).
(b) To show (b) implies (a), consider the open statement (the correct choice of an open statement is half the battle in induction proofs) \( Q(n), n \in A \) to be,

If \( S \subset A \) with \( m \in S \) with \( m \leq n \), then \( S \) has a minimal element.

Show that \( Q(N) \) is true and \( Q(n) \Rightarrow Q(n + 1) \) for any \( n \in A \) and deduce (a).

(2) Here is another variation of induction. So, let \( N, A \) be as above and let \( P(n) \) be an open statement for \( n \in A \). Assume \( P(N) \)
is true, \( P(N + 1) \) is true and \( P(n) \Rightarrow P(n + 2) \) is true for all \( n \in A \). Show that \( P(n) \) is true for all \( n \in A \).

(3) Show that \( \sum_{k=0}^{n} \binom{n}{k} = 2^n \) for any \( n \in \mathbb{N} \).

(4) Let \( n \in \mathbb{N} \) and let \( s(n) \) be the sum of its digits (written the usual way). Show that \( n \equiv s(n) \mod 9 \).

(5) Show that for any \( n \in \mathbb{N}, 4^n \not\equiv 2 \mod 9 \).