

HOMEWORK 6, DUE OCT 18, 2017

You may use any result that we proved in class. If you can, quote them by name or statement or by some other identifying feature.

- (1) Decide whether the following are true. If true, give a proof and if false, give a counterexample to illustrate your reasoning.
 - (a) If A, B are subsets of Ω and $A \cap B = \emptyset$, then $A = \emptyset$ or $B = \emptyset$.
 - (b) If A is not contained in B and $C \subset B$, then A is not contained in C .
- (2) We generalize the notions introduced in class for possibly infinite collections as follows. Let Λ be a non-empty set, possibly infinite. We denote by Greek letters, elements of Λ .

Assume Ω is a fixed set and we are given subsets $A_\alpha \subset \Omega$ for each $\alpha \in \Lambda$. (We have *indexed* the collection of subsets by Λ). As usual define,

$$\cup_{\alpha \in \Lambda} A_\alpha = \{a \in \Omega \mid a \in A_\alpha \text{ for some } \alpha \in \Lambda\},$$

$$\cap_{\alpha \in \Lambda} A_\alpha = \{a \in \Omega \mid a \in A_\alpha \text{ for all } \alpha \in \Lambda\}.$$

- (a) Show that if $B \subset \Omega$, then $B \cap (\cup_{\alpha \in \Lambda} A_\alpha) = \cup_{\alpha \in \Lambda} (B \cap A_\alpha)$.
 - (b) Show that $(\cup_{\alpha \in \Lambda} A_\alpha)^c = \cap_{\alpha \in \Lambda} A_\alpha^c$.
 - (c) For this problem, we use $\Lambda = \mathbb{N}$ (so instead of Greek letters, we use English letters) and Ω to be the open interval $(0, 1)$. Write down a collection of subsets $A_n, n \in \mathbb{N}$ of $(0, 1)$ such that for any $j, k \in \mathbb{N}$, $A_j \cap A_k \neq \emptyset$, but $\cap_{n \in \mathbb{N}} A_n = \emptyset$. (Hint: A real number x is positive is the same as saying $x \geq \frac{1}{n}$ for some $n \in \mathbb{N}$.)
- (3) Define as we did in class, for two subsets A, B of Ω ,

$$A - B = \{a \in \Omega \mid a \in A, \text{ but } a \notin B\}.$$

Show that for subsets A, B, C of Ω , $(A - B) - C = A - (B \cup C)$.

- (4) Define for two sets A, B as above,

$$A + B = (A - B) \cup (B - A).$$

Show that $A + A = \emptyset$ and $A + \emptyset = A$ for any $A \subset \Omega$.

- (5) If $X, Y \subset A$ and B any other set, show that $(X \cap Y) \times B = (X \times B) \cap (Y \times B)$, all considered as subsets of $A \times B$.