Math 131, Spring 2004
Quiz #4, Discussion Section A (Thursday, 11:00-12:00)

Quiz problems should be solved using the methods discussed in this course. A calculator is not permitted. To receive full credit, show enough work to make it clear how you got your answer.

Name: Answer Key ID#

1. Where is the tangent line to \( f(x) = 2x^3 + 3x^2 - 12x + e^x \) horizontal?

\[
\begin{align*}
f(x) &= 2x^3 + 3x^2 - 12x + e^x \\
f'(x) &= 6x^2 + 6x - 12
\end{align*}
\]

So \( f'(x) = 0 \) \( \iff \) \( 6x^2 + 6x - 12 = 0 \) \( \iff \) \( x^2 + x - 2 = 0 \) \( \iff \) \( (x+2)(x-1) = 0 \) \( \iff \) \( x = -2 \) or \( x = 1 \)

So the tangent line is horizontal at \( x = -2 \) & \( x = 1 \).

2. Sketch a graph of a function with the following properties:

(i). \( f(0) = 0 \) and \( f'(-1) = f'(1) = f'(3) = 0 \)
(ii). \( f'' > 0 \) on \( (-\infty, 0) \) and \( (2, \infty) \) and \( f'' < 0 \) on \( (0, 2) \).
(iii). \( f' > 0 \) on \( (-1, 1) \) and \( (3, \infty) \) and \( f' < 0 \) on \( (-\infty, -1) \) and \( (1, 3) \)

(i) \( \Rightarrow \) tan. line horizontal at \( x = -1, x = 1, \) & \( x = 3 \)

(ii) \( \Rightarrow \) f c.c. ↑ on \( (-\infty, 0) \) & \( (2, \infty) \) \( \Rightarrow \) f has an inflection pt at \( x = 0 \) & \( x = -2 \)

(iii) \( \Rightarrow \) f increasing on \( (-1, 1), (3, \infty) \) & decreasing on \( (-\infty, -1), (1, 3) \)

So f has a min at \( x = -1 \), a max at \( x = 1 \), & a min at \( x = 3 \)

\[\text{Graph of the function.}\]
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Name: Answer Key         ID#

1. A particle is moving along the x-axis. At time $t$ seconds, its velocity is $v(t) = \sqrt{2t+3}$. When $t = 2s$, its position is 9 meters. Use linear approximation to estimate the position of the particle $\frac{1}{8}$s later. Is this estimate an overestimate or an underestimate? Why?

Let $p(t)$ denote the position function. We are given 
$p(2) = 9$. Since $p'(2) = v(2) = \sqrt{9} = 3$. So the tan. line to $p(t)$ at 2 is $y - 9 = 3(x-2)$ or $y = 3x + 3$. So our linear approx. function is $L(x) = 3x + 3$. We estimate $p(2.25) \approx L(2.25) = 3(2.25) + 3 = 9.75$.

Now $p'(t) = v(t)$ is increasing at $t = 2$, so $p''(t) > 0$ at $t = 2$. Therefore, $p$ is concave up at $t = 2$. So the tangent line lies under the curve & our estimate is an underestimate.

2. Draw a graph of a function such that the first derivative changes sign exactly once and the second derivative is always negative. To help you with this, please answer the following questions.

   a) If $f'$ changes sign at $x = a$, then what can we say about $f(a)$?

      $f$ has a horizontal tangent at $x = a$.

   b) If $f''$ is always negative, then what can we say about $f$?

      $f$ is always concave down

   c) Sketch an example of a function with the above two properties.
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Name: ____________________  ID #: ____________________

1. Please answer the following true/false questions, and explain your reasoning. (Your explanation can consist of a picture.)

a) True/False: It is possible for a function \( f \) to be concave down and have its derivative \( f' \) be an increasing function.

\[
\begin{align*}
&f \text{ c.c. } \downarrow \Rightarrow f'' < 0 \\
&f' \text{ inc. } \Rightarrow f'' > 0
\end{align*}
\]

So false

b) True/False: If \( f'' \) changes sign exactly once, then \( f \) must be a decreasing function.

False : 

\[
\begin{array}{c}
\text{False :} \\
\text{For this } f, \ f \text{ is c.c. } \downarrow \text{ on } (-\infty, 0) \text{ & c.c. } \uparrow \text{ on } (0, \infty), \text{ so } f'' \text{ changes sign exactly once at } x=0, \text{ but } f \text{ is not decreasing.}
\end{array}
\]

c) True/False: If \( f' \) changes sign at \( x = a \) and \( f'' \) is always positive, then \( x = a \) is a local minimum.

\[
\begin{align*}
f'' > 0 & \Rightarrow f \text{ c.c. } \uparrow \\
& \Rightarrow a \text{ is a local min.}
\end{align*}
\]

So True.

2. Compute the following:

a) (1 point) \( \frac{d}{dx}(x^e) = ex^{e-1} \)

b) (2 points) \( \frac{d}{dx} \left( \frac{4x^3 - 3x - 3\sqrt{x}}{x} \right) = \frac{d}{dx} \left( 4x - 3 + 3x^{-1/2} \right) = 4 - \frac{3}{2}x^{-3/2} \)