Solutions

to

Math 132 Midterm Exam 1

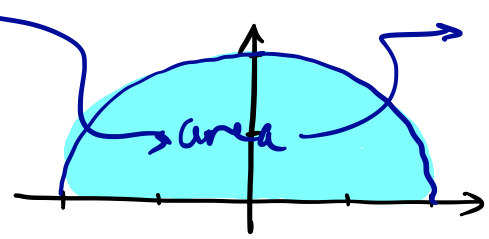
This exam consists of 10 multiple choice (machine-graded) problems, worth 3 points each (for a total of 30 points), and 2 pages of written (hand-graded) problems, worth a total of 20 points. No 3x5 cards or calculators are allowed.

Part I: Multiple choice problems

You will need a pencil to mark your card. If you do not have one, please ask your proctor. Write your **ID number** (not your SS number) on the six blank lines on the top of your answer card, using one blank for each digit. **Shade in the corresponding boxes below.** Also **print your name at the top of your card.**

(1) Find the general antiderivative of $\sin(x)$.
(A) $\sin(x) + C$
(B) $\cos(x) + C$
(C) $-\sin(x) + C$
(D) $-\cos(x) + C$

(2) Use a picture to find $\int_{-2}^{2} \sqrt{4 - x^2}dx$. =
(A) 0
(B) π
(C) 2π
(D) 3π
(E) 4π



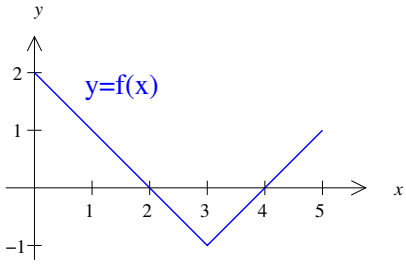
$= \pi r^2 \cdot \frac{1}{2}$

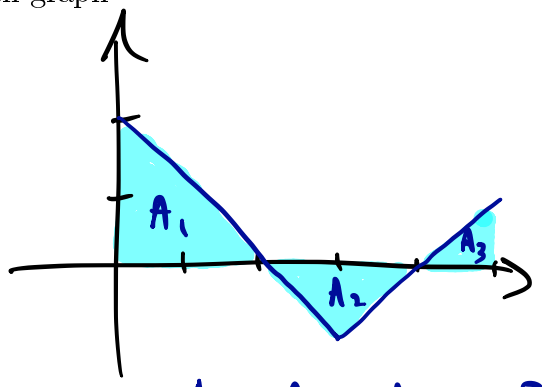
$= \pi \cdot 2^2 \cdot \frac{1}{2}$

$= 2\pi$

(3) Which is an antiderivative of $2^x - e^x$?
(A) $2^x - e^x$
(B) $2^x \ln 2 - e^x$
(C) $\frac{2^x}{\ln 2} - e^x$
(D) $2^{x+1} - e^{x+1}$

(4) Determine $\int_0^5 f(x)dx$ for the function with graph





integral $= A_1 - A_2 + A_3 = 2 - 1 + \frac{1}{2} = \frac{3}{2}$

(A) $\frac{7}{2}$
(B) 3
(C) $\frac{5}{2}$
(D) 2
(E) $\frac{3}{2}$
(F) 1

(5) Find an antiderivative of $\frac{1+3x+5x^2}{\sqrt{x}}$.

(A) $\frac{x+\frac{3}{2}x^2+\frac{5}{3}x^3}{\sqrt{x}}$

(B) $\frac{2x+2x^2+2x^3}{\sqrt{x}}$

(C) $\frac{x+x^2+x^3}{\sqrt{x}}$

(D) $\frac{-2x+6x^2+\frac{10}{3}x^3}{\sqrt{x}}$

$= x^{-\frac{1}{2}} + 3x^{\frac{1}{2}} + 5x^{\frac{3}{2}}$

$\downarrow$ ANTI

$\frac{x^{1/2}}{1/2} + \frac{\cancel{3}x^{3/2}}{\cancel{3}/2} + \frac{\cancel{5}x^{5/2}}{\cancel{5}/2} = 2x^{1/2} + 2x^{3/2} + 2x^{5/2} = \frac{2x+2x^2+2x^3}{\sqrt{x}}$

(6) Which are the best possible upper and lower bounds U and L that one can obtain on the area A under the graph of $y = 4^x$ from $x = 1$ to $x = 3$, using four rectangles of equal width?

(A) $L = 10$, $U = 60$

(B) $L = 20$, $U = 80$

(C) $L = 30$, $U = 60$

(D) $L = 30$, $U = 80$

(E) $L = 40$, $U = 80$

(F) $L = 40$, $U = 60$

$L = \frac{1}{2}\cdot 4^1 + \frac{1}{2}\cdot 4^{3/2} + \frac{1}{2}\cdot 4^2 + \frac{1}{2}\cdot 4^{5/2}$

$= \frac{1}{2}(4+8+16+32) = 30$

$U = \frac{1}{2}\cdot 4^{3/2} + \frac{1}{2}\cdot 4^2 + \frac{1}{2}\cdot 4^{5/2} + \frac{1}{2}\cdot 4^3$

$= \frac{1}{2}(8+16+32+64) = 60$

(7) Find a region whose area is equal to $\lim_{n\to\infty} \frac{2}{n}\sum_{i=1}^{n} e^{1-\frac{2i}{n}}$. (Here $(f(x),[a,b])$ means area under the graph of $y = f(x)$, from $x = a$ to $x = b$.)

(A) The region under the graph of $y = e^{-x}$ from $x = 1$ to $x = 3$.

(B) The region under the graph of $y = 2e^{-x}$ from $x = -1$ to $x = 1$.

(C) The region under the graph of $y = e^{-2x}$ from $x = -1$ to $x = 1$.

(D) The region under the graph of $y = e^{-x}$ from $x = -1$ to $x = 1$.

sample point:

$x_i = -1 + \frac{2i}{n}$

$\Rightarrow e^{-x_i} = e^{1-\frac{2i}{n}}$.

(8) If $\int_3^2 f(x)dx = 1$, $\int_2^7 f(x)dx = 4$, and $\int_5^7 f(x)dx = 2$, find $\int_3^5 f(x)dx$. [Remark: $\int_3^2$ is not a typo.]

(A) -2

(B) -1

(C) 0

(D) 1

(E) 2

(F) 3

$\int_3^5 = \int_2^7 - \int_2^3 - \int_5^7 = \int_2^7 + \int_3^2 - \int_5^7$

$= 4 + 1 - 2 = 3$

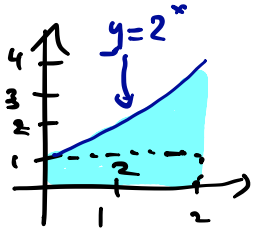
(9) Which bound is not correct?

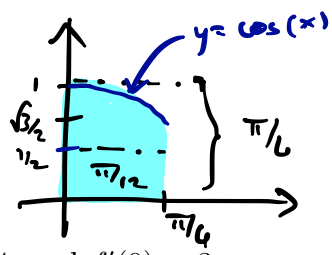
(A) $\int_0^2 2^x dx \le 4$

(B) $\int_0^2 2^x \ge 2$

(C) $\int_0^{\frac{\pi}{6}} \cos(x)dx \le \frac{\pi}{6}$

(D) $\int_0^{\frac{\pi}{6}} \cos(x)dx \ge \frac{\pi}{12}$





So by process of elimination, (A) is the only one that can be wrong... and indeed,

$\int_0^2 2^x dx = \frac{2^2-2^0}{\ln 2} = \frac{3}{\ln 2} \approx 4.32$

(10) Find $f(x)$ such that $f''(x) = \cos(x) - \sin(x)$, $f(0) = 4$, and $f'(0) = 3$.

(A) $-\cos(x) + \sin(x) + 4 + 3x$

(B) $4\cos(x) + 3\sin(x)$

(C) $-4\cos(x) + 3\sin(x)$

(D) $-\cos(x) + \sin(x) + 5 + 2x$

general double antiderivative of $\cos(x) - \sin(x)$ is

$-\cos(x) + \sin(x) + Ax + B = f(x)$

$\Rightarrow f'(0) = \sin(0) + \cos(0) + A = 1 + 1 \ (= 3)$

$f(0) = -\cos(0) + \sin(0) + B = B - 1 \ (= 4)$

$\Rightarrow A = 2,\ B = 5$

Part II: Hand-graded problems

This part has two pages. Show all the work you want graded for each problem in the space provided. Please **print your name at the top of each page.**

(1) (a) [6 points] Compute the antiderivative $F(x)$ of $x - \frac{1}{x}$ with $F(1) = 0$.

- general antiderivative $= \frac{x^2}{2} - \ln(x) + C = F(x)$
- then $0 = F(1) = \frac{1^2}{2} - \ln(1) + C = \frac{1}{2} - 0 + C$
  $= \frac{1}{2} + C \implies C = -\frac{1}{2}$
- conclude that $\boxed{F(x) = \frac{x^2}{2} - \ln(x) - \frac{1}{2}}$

(b) [4 points] Compute an antiderivative of $\frac{4x^2-2x+1}{x^2+1}$.

$$\frac{4x^2 - 2x + 1}{x^2+1} = \frac{4x^2 + 4 - 2x - 3}{x^2+1} = 4 - \frac{2x}{x^2+1} - \frac{3}{x^2+1}$$

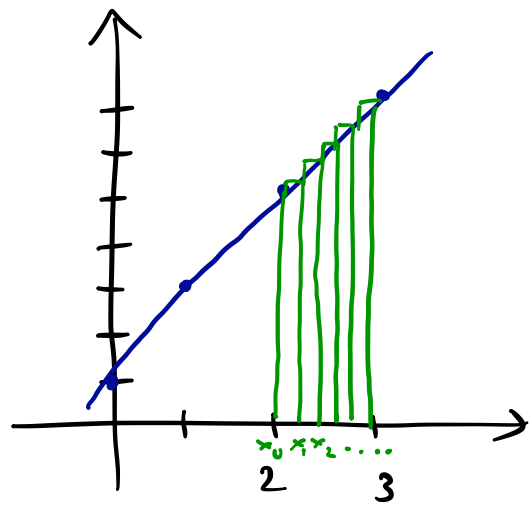
$$\xrightarrow[\text{ANTI}]{} \boxed{4x - \ln(x^2+1) - 3\arctan(x)}.$$

(2) (a) [6 points] Represent the area under the graph of $y = 2x + 1$ from $x = 2$ to $x = 3$ as the limit of a sum, and draw the corresponding picture.

$$A = \lim_{n\to\infty} \frac{1}{n} \sum_{i=1}^{n} (2x_i + 1)$$

$$= \lim_{n\to\infty} \frac{1}{n} \sum_{i=1}^{n} \left(2\left(2 + \frac{i}{n}\right) + 1\right)$$

$$= \lim_{n\to\infty} \frac{1}{n} \sum_{i=1}^{n} \left(5 + \frac{2i}{n}\right)$$



$x_i = 2 + \frac{i}{n}$

(b) [4 points] Use a formula (not the Fundamental Theorem of Calculus) to calculate the limit from part (a).

$$\sum_{i=1}^{n} \left(5 + \frac{2i}{n}\right) = \sum_{i=1}^{n} 5 + \frac{2}{n} \sum_{i=1}^{n} i = 5n + \frac{\cancel{2}}{\cancel{n}} \cdot \frac{\cancel{n}(n+1)}{\cancel{2}}$$

$$= 5n + n + 1 = 6n + 1$$

$$\Rightarrow A = \lim_{n\to\infty} \frac{1}{n}(6n + 1) = \lim_{n\to\infty} \left(6 + \frac{1}{n}\right) = 6.$$