Solutions to

Math 132 Midterm Exam 1

This exam consists of 10 multiple choice (machine-graded) problems, worth 3 points each (for a total of 30 points), and 2 pages of written (hand-graded) problems, worth a total of 20 points. No 3x5 cards or calculators are allowed.

Part I: Multiple choice problems

You will need a pencil to mark your card. If you do not have one, please ask your proctor. Write your ID number (not your SS number) on the six blank lines on the top of your answer card, using one blank for each digit. Shade in the corresponding boxes below. Also print your name at the top of your card.

(1) Find the general antiderivative of \( \sin(x) \).
   (A) \( \sin(x) + C \)
   (B) \( \cos(x) + C \)
   (C) \( -\sin(x) + C \)
   (D) \( -\cos(x) + C \)  
   \( \boxed{\text{(D)}} \)

(2) Use a picture to find \( \int_{-2}^{2} \sqrt{4 - x^2} \, dx \).
   (A) 0
   (B) \( \pi \)
   (C) \( 2\pi \)
   (D) \( 3\pi \)
   (E) \( 4\pi \)  
   \( \boxed{\text{(C)}} \)

(3) Which is an antiderivative of \( 2^x - e^x \)?
   (A) \( 2^x - e^x \)
   (B) \( 2^x \ln 2 - e^x \)
   (C) \( \frac{2^x}{\ln 2} - e^x \)
   (D) \( 2^{x+1} - e^{x+1} \)
   \( \boxed{\text{(C)}} \)

(4) Determine \( \int_{0}^{5} f(x) \, dx \) for the function with graph

   \( \int_{0}^{5} f(x) \, dx = \) \( \frac{1}{2} \)
   \( \boxed{\text{(B)}} \)

\[ \text{infty} = A_1 - A_2 + A_3 = 2 - 1 + \frac{1}{2} = \frac{3}{2} \]
(5) Find an antiderivative of \( \frac{1+3x+5x^2}{\sqrt{x}} \).

(A) \( \frac{x^2+\frac{3}{2}x^2+\frac{5}{2}x^3}{\sqrt{x}} \)

(B) \( \frac{2x^2+2x^3+2x^4}{\sqrt{x}} \)

(C) \( \frac{x^2+x^2+2x^3}{\sqrt{x}} \)

(D) \( \frac{-2x+6x^3+10x^3}{\sqrt{x}} \)

(6) Which are the best possible upper and lower bounds \( U \) and \( L \) that one can obtain on the area \( A \) under the graph of \( y = 4^x \) from \( x = 1 \) to \( x = 3 \), using four rectangles of equal width?

(A) \( L = 10, \ U = 60 \)

(B) \( L = 20, \ U = 80 \)

(C) \( L = 30, \ U = 60 \)

(D) \( L = 30, \ U = 80 \)

(E) \( L = 40, \ U = 80 \)

(F) \( L = 40, \ U = 60 \)

(7) Find a region whose area is equal to \( \lim_{n \to \infty} \frac{1}{n} \sum_{i=1}^{n} e^{1-\frac{2i}{n}} \). (Here \((f(x), [a, b])\) means area under the graph of \( y = f(x) \), from \( x = a \) to \( x = b \).)

(A) The region under the graph of \( y = e^{-x} \) from \( x = 1 \) to \( x = 3 \).

(B) The region under the graph of \( y = 2e^{-x} \) from \( x = -1 \) to \( x = 1 \).

(C) The region under the graph of \( y = e^{-2x} \) from \( x = -1 \) to \( x = 1 \).

(D) The region under the graph of \( y = e^{-x} \) from \( x = -1 \) to \( x = 1 \).

(8) If \( \int_{3}^{5} f(x) \, dx = 1 \), \( \int_{2}^{7} f(x) \, dx = 4 \), and \( \int_{5}^{7} f(x) \, dx = 2 \), find \( \int_{3}^{5} f(x) \, dx \). [Remark: \( \int_{3}^{5} \) is not a typo.]

(A) \(-2\)

(B) \(-1\)

(C) \(0\)

(D) \(1\)

(E) \(2\)

(F) \(3\)

(9) Which bound is not correct?

(A) \( \int_{0}^{2} 2x \, dx \leq 4 \)

(B) \( \int_{0}^{2} x \geq 2 \)

(C) \( \int_{0}^{\pi} \cos(x) \, dx \leq \frac{\pi}{2} \)

(D) \( \int_{0}^{\frac{\pi}{2}} \cos(x) \, dx \geq \frac{\pi}{2} \)

(10) Find \( f(x) \) such that \( f''(x) = \cos(x) - \sin(x) \), \( f(0) = 4 \), and \( f'(0) = 3 \).

(A) \(-\cos(x) + \sin(x) + 4 + 3x\)

(B) \(4\cos(x) + 3\sin(x)\)

(C) \(-4\cos(x) + 3\sin(x)\)

(D) \(-\cos(x) + \sin(x) + 5 + 2x\)
PART II: HAND-GRADED PROBLEMS

This part has two pages. Show all the work you want graded for each problem in the space provided. Please print your name at the top of each page.

(1) (a) [6 points] Compute the antiderivative of $x - \frac{1}{x}$ with $F(1) = 0$.

- General antiderivative: $F(x) = \frac{x^2}{2} - \ln(x) + C = F(x)$

- Then $F(1) = \frac{1^2}{2} - \ln(1) + C = \frac{1}{2} - 0 + C = \frac{1}{2} + C \implies C = -\frac{1}{2}$

- Conclude that $F(x) = \frac{x^2}{2} - \ln(x) - \frac{1}{2}$

(b) [4 points] Compute an antiderivative of $\frac{4x^2 - 2x + 1}{x^2 + 1}$.

\[
\frac{4x^2 - 2x + 1}{x^2 + 1} = \frac{4x^2 + 4 - 2x - 3}{x^2 + 1} = 4 - \frac{2x}{x^2 + 1} - \frac{3}{x^2 + 1}
\]

$\implies \quad \text{ANT} \quad 4x - \ln(x^2 + 1) - 3 \arctan(x)$. 
(2) (a) [6 points] Represent the area under the graph of \( y = 2x + 1 \) from \( x = 2 \) to \( x = 3 \) as the limit of a sum, and draw the corresponding picture.

\[
A = \lim_{n \to \infty} \frac{1}{n} \sum_{i=1}^{n} \left( 2x_i + 1 \right)
\]

\[
= \lim_{n \to \infty} \frac{1}{n} \sum_{i=1}^{n} \left( 2 \left( 2 + \frac{i}{n} \right) + 1 \right)
\]

\[
= \lim_{n \to \infty} \frac{1}{n} \sum_{i=1}^{n} \left( 5 + \frac{2i}{n} \right)
\]

(b) [4 points] Use a formula (not the Fundamental Theorem of Calculus) to calculate the limit from part (a).

\[
\sum_{i=1}^{n} \left( 5 + \frac{2i}{n} \right) = \sum_{i=1}^{n} 5 + \frac{2}{n} \sum_{i=1}^{n} i = 5n + \frac{2}{n} \cdot \frac{n(n+1)}{2}
\]

\[
= 5n + n + 1 = 6n + 1
\]

\[
\Rightarrow A = \lim_{n \to \infty} \frac{1}{n} \left( 6n + 1 \right) = \lim_{n \to \infty} \left( 6 + \frac{1}{n} \right) = 6.
\]