

Lecture 1: Antidifferentiation

- Course website: <http://www.math.wustl.edu/~matkerr/132>
- Exam 1: Is on Tuesday, Jan. 30th, and will cover only sections 4.9, 5.1, and 5.2.

————— • —————

Taking antiderivative is the opposite of taking derivative — you go backwards. Given $f(x)$, you try to find $F(x)$ so that

$$F'(x) = f(x).$$

Then $F(x)$ is called a particular antiderivative of $f(x)$.

Suppose you have two, F_1 and F_2 :

$$F_1'(x) = f(x) \quad \text{and} \quad F_2'(x) = f(x). \quad \text{Then}$$

$$\frac{d}{dx}(F_1 - F_2) = F_1' - F_2' = f - f = 0$$

$\Rightarrow F_1 - F_2$ is a constant C , i.e.

$$F_1 = F_2 + C.$$

Therefore all antiderivatives of a given $f(x)$ differ by a constant. If $F(x)$ is a particular antiderivative, then the general antiderivative is

$$F(x) + C.$$

EXAMPLES

- POWER RULE BACKWARDS:

$$\frac{d}{dx} x^n = n x^{n-1}$$

$$\Rightarrow \frac{d}{dx} x^{n+1} = (n+1) x^n$$

$$\Rightarrow \frac{d}{dx} \left(\frac{x^{n+1}}{n+1} \right) = x^n$$

↙
antiderivative
of x^n

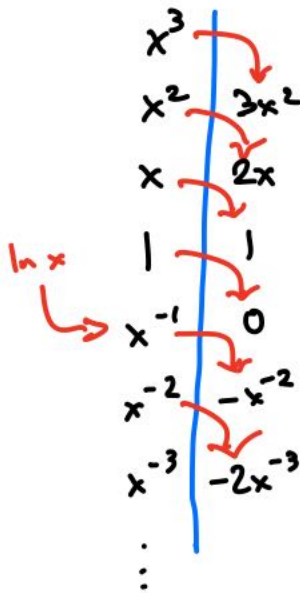
So for $f(x) = x^n$, $F(x) = \frac{x^{n+1}}{n+1} + C$.

for general

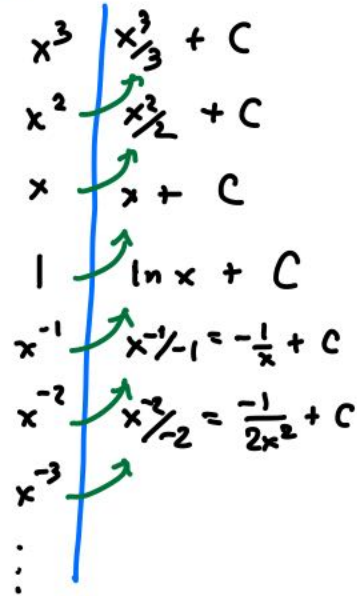
works as long as $n \neq -1$

— what goes wrong there?

DIFF.



ANTI DIFF.



So $f(x) = \frac{1}{x^3} \rightarrow F(x) = -\frac{1}{2x^2} \dots$

but $f(x) = \frac{1}{x} \rightarrow F(x) = \ln(x)$, if $x > 0$.

(Actually, for $x \neq 0$, it's $\ln|x|$.)

• EXP AND TRIG :

You'll remember that $\frac{d}{dx} e^{ax} = a e^{ax}$

So $\frac{d}{dx} \frac{1}{a} e^{ax} = e^{ax}$,

And $f(x) = e^{ax} \xrightarrow{\text{ANTI DIFF.}} F(x) = \frac{1}{a} e^{ax}$.

Likewise, we get

$$f(x) = \sin(ax) \longrightarrow F(x) = -\frac{1}{a} \cos(ax)$$

$$f(x) = \cos(ax) \longrightarrow F(x) = \frac{1}{a} \sin(ax).$$

What about (say)

$$f(x) = 3^x ?$$

- LINEARITY: If $f \rightarrow F$ & $g \rightarrow G$ are antiderivatives, so is $af \pm bg \rightarrow aF \pm bG$, Same as with derivatives.

- INVERSE TRIG:

You may remember that (for instance)

$$\frac{d}{dx} \arcsin(x) = \frac{1}{\sqrt{1-x^2}}$$

← ANTI

$$\frac{d}{dx} \arctan(x) = \frac{1}{1+x^2}$$

← ANTI

The second antiderivative comes up in unexpected places:

What's the antiderivative of $\frac{1-x^2}{1+x^2}$? Write

$$\frac{1-x^2}{1+x^2} = \frac{2}{1+x^2} - \frac{1+x^2}{1+x^2} = 2 \cdot \frac{1}{1+x^2} - 1 \longrightarrow 2 \arctan(x) - x.$$

ANTI

Anyway, you have a bunch of problems asking you for general antiderivatives, and a bunch asking for solutions of "initial value problems", which involve finding a particular antiderivative (or 2nd antiderivative, etc.) satisfying some conditions (like $f(2)=3$).

Ex / $\frac{dy}{dx} = x(1+x)$, $x > 0$. (This says "the derivative of y is $x(1+x)$ ".) $F'(x) = f(x)$.

So y is antiderivative of $f(x) = x(x+1) = x^2 + x$.

Power rule backwards gives

$$y = F(x) = \frac{x^3}{3} + \frac{x^2}{2} + C \quad \text{for the general sol'n.}$$

Now if the problem adds an initial condition, like " $y = 6$ when $x = 0$ ", we need

$$6 = F(0) = \frac{0^3}{3} + \frac{0^2}{2} + C$$

$\Rightarrow C = 6$, and the solution to this "initial value problem" is just

$$y = \frac{x^3}{3} + \frac{x^2}{2} + 6.$$

Check: $\frac{dy}{dx} = \frac{3x^2}{3} + \frac{2x}{2} = x^2 + x$ ✓ //

Ex / Try $\frac{dy}{dx} = \frac{1}{2\sqrt{x}}$, for $x \geq 1$ with $y = 3$
when $x = 4$. //