Lecture 1 : Antidifferentiation

- Course website : http://www.math.wustl.edu/~matkerr/132
- Exam 1 : Is on Tuesday, Jan. 30th, and will cover only sections 4.9, 5.1, and 5.2.

Taking antiderivative is the opposite of taking derivative — you go backwards. Given $f(x)$, you try to find $F(x)$ so that

$$F'(x) = f(x).$$

Then $F(x)$ is called a particular antiderivative of $f(x)$.

Suppose you have two, F_1 and F_2:

$F_1'(x) = f(x)$ and $F_2'(x) = f(x)$. Then

$$\frac{d}{dx}(F_1 - F_2) = F_1' - F_2' = f - f = 0$$

$\Longrightarrow$ $F_1 - F_2$ is a constant C, i.e.

$$F_1 = F_2 + C.$$

Therefore all antiderivatives of a given $f(x)$ differ by a constant. If $F(x)$ is a particular antiderivative, then the <u>general antiderivative</u> is

$$F(x) + C.$$

EXAMPLES

- <u>POWER RULE BACKWARDS</u>:

$$\frac{d}{dx} x^n = n x^{n-1}$$

$$\Rightarrow \frac{d}{dx} x^{n+1} = (n+1) x^n$$

$$\Rightarrow \frac{d}{dx} \frac{x^{n+1}}{n+1} = x^n$$

antiderivative of x^n

So for $f(x) = x^n$, $\quad F(x) = \frac{x^{n+1}}{n+1} \underbrace{+ C}_{\text{for general}}$.

Works as long as $n \neq -1$

— what goes wrong there?

DIFF.

	x^3	
	x^2	$3x^2$
	x	$2x$
$\ln x$	1	1
	x^{-1}	0
	x^{-2}	$-x^{-2}$
	x^{-3}	$-2x^{-3}$
	$\vdots$	

ANTIDIFF.

x^3	$x^3/3 + C$
x^2	$x^2/2 + C$
x	$x + C$
1	$\ln x + C$
x^{-1}	$x^{-1}/-1 = -\frac{1}{x} + C$
x^{-2}	$x^{-2}/-2 = \frac{-1}{2x^2} + C$
x^{-3}	
$\vdots$	

So $f(x) = \frac{1}{x^3} \longrightarrow F(x) = \frac{-1}{2x^2}$ ---

but $f(x) = \frac{1}{x} \longrightarrow F(x) = \ln(x)$, if $x>0$.

(Actually, for $x \neq 0$, it's $\ln|x|$.)

- <u>EXP AND TRIG</u>:

You'll remember that $\frac{d}{dx} e^{ax} = a e^{ax}$

So $\frac{d}{dx} \frac{1}{a} e^{ax} = e^{ax}$,

and $f(x) = e^{ax} \xrightarrow[\text{ANTIDIFF.}]{} F(x) = \frac{1}{a} e^{ax}$.

Likewise, we get

$$f(x) = \sin(ax) \longrightarrow F(x) = -\frac{1}{a}\cos(ax)$$

$$f(x) = \cos(ax) \longrightarrow F(x) = \frac{1}{a}\sin(ax).$$

What about (say)

$$f(x) = 3^x \quad ?$$

- LINEARITY: If $f \longrightarrow F$ & $g \longrightarrow G$ are antiderivatives, so is $af \pm bg \longrightarrow aF \pm bG$, same as with derivatives.

- INVERSE TRIG:

You may remember that (for instance)

$$\frac{d}{dx}\arcsin(x) = \frac{1}{\sqrt{1-x^2}}$$

ANTI

$$\frac{d}{dx}\arctan(x) = \frac{1}{1+x^2}$$

ANTI

The second antiderivative comes up in unexpected places: what's the antiderivative of $\frac{1-x^2}{1+x^2}$? Write

$$\frac{1-x^2}{1+x^2} = \frac{2}{1+x^2} - \frac{1+x^2}{1+x^2} = 2\cdot\frac{1}{1+x^2} - 1 \underset{\text{ANTI}}{\longrightarrow} 2\arctan(x) - x.$$

Anyway, you have a bunch of problems asking you for <u>general</u> antiderivatives, and a bunch asking for solutions of "initial value problems", which involve finding a particular antiderivative (or 2^{nd} antiderivative, etc.) satisfying some conditions (like $f(2)=3$).

Ex/ $\frac{dy}{dx} = x(1+x)$, $x>0$. (This says "the derivative of $\underbrace{y}_{F(x)}$ is $\underbrace{x(1+x)}_{f(x)}$.") $F'(x)=f(x)$.

So y is antiderivative of $f(x) = x(x+1) = x^2+x$. Power rule backwards gives

$y = F(x) = \frac{x^3}{3} + \frac{x^2}{2} + C$ for the general sol'n.

Now if the problem adds an initial condition, like "$y=6$ when $x=0$", we need

$$6 = F(0) = \frac{0^3}{3} + \frac{0^2}{2} + C$$

$\Rightarrow C = 6$, and the solution to this "initial value problem" is just

$$y = \frac{x^3}{3} + \frac{x^2}{2} + 6 .$$

<u>Check</u>: $\frac{dy}{dx} = \frac{3x^2}{3} + \frac{2x}{2} = x^2 + x$ ✓

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Ex/ Try $\frac{dy}{dx} = \frac{1}{2\sqrt{x}}$, for $x \geq 1$ with $y = 3$ when $x = 4$.

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