FINAL REVIEW

Part I : Matrix Algebra

Apostol Ch. 2-5

Part II : Differential Equations

Apostol Ch. 6-7

Part III : Multivariable Differential Calculus

Apostol Ch. 8-9

Part IV : Multivariable Integral Calculus

Apostol Ch. 10-12

- using RREF to solve linear systems, determine independence of vectors & rank of matrix / linear transformation, and compute inverses
- matrix of a linear transformation w.r.t. a basis, change of basis, compositions
- methods for computing determinants: RREF, Laplace expansion, etc.; interpretation as expansion factor, product of eigenvalues
- eigenvalues, eigenvectors, characteristic polynomials, diagonalization
- spectral theorem (orthogonal diagonalizability for symmetric real matrices), generalization to self-adjoint / unitary / skew-adjoint operators; quadratic forms
- Cayley-Hamilton (A satisfies its own characteristic polynomial: $p_A(A) = 0$)

Part I : Matrix Algebra

Apostol Ch. 2-5

Part II : Differential Equations

Apostol Ch. 6-7

Part III : Multivariable Differential Calculus

Apostol Ch. 8-9

Part IV : Multivariable Integral Calculus

Apostol Ch. 10-12

- homogeneous linear ODE with constant coeffs.
  - get a basis of solutions by factoring its characteristic polynomial
- inhomogeneous linear ODE with constant coeffs.
  - to get a particular solution, use "reduction to first-order", "annihilator method", or "variation of parameters" (which works for analytic coeffs. provided you have the basis of homogeneous solutions)
- power-series solutions, indicial equations, & method of Frobenius (only as described in the book) for homogeneous linear ODE with analytic coefficients
- solving systems of the form (A $n \times n$, constant)

$$\vec{x}'(t) = A\vec{x}(t),$$

esp. by diagonalization. (The reason Cayley-Hamilton was introduced was to deal with the nondiagonalizable case. While I want you to know C-H, this application won't be on the exam.)

Part I : Matrix Algebra

Apostol Ch. 2 – 5

Part II : Differential Equations

Apostol Ch. 6 – 7

Part III : Multivariable Differential Calculus

Apostol Ch. 8 – 9

Part IV : Multivariable Integral Calculus

Apostol Ch. 10 – 12

- limits, directional & partial derivatives, and differentiability of functions of several variables
- Chain rule and Jacobians
- bit of PDE, e.g. parts 1 & 4 of Lecture 29
- local extrema / saddles, Hessian matrix (2nd derivative test)
- optimization : stationary points; constraints and Lagrange multipliers (be prepared for multiple constraints)

Part I : Matrix Algebra

Apostol Ch. 2-5

Part II : Differential Equations

Apostol Ch. 6-7

Part III : Multivariable Differential Calculus

Apostol Ch. 8-9

Part IV : Multivariable Integral Calculus

Apostol Ch. 10-12

- line integrals : $\int_C f\,ds$, $\int_C \vec{F}\cdot d\vec{r}$, $\int_C P\,dx + Q\,dy + \cdots$ (convert to integral on $[a,b]$ by using a parametrization $\vec{r}$; ds becomes $\|\vec{r}\,'(t)\|\,dt$, $d\vec{r}$ becomes $\vec{r}\,'(t)\,dt$, etc.)
- FTC II : $\int_C \vec{\nabla} f \cdot d\vec{r} = f(\vec{b}) - f(\vec{a})$ $\vec{a} \xrightarrow{C} \vec{b}$

  FTC I : $\vec{F}$ conservative $\Rightarrow \vec{\nabla} \underbrace{\int_{\vec{x}_0}^{\vec{x}} \vec{F}\cdot d\vec{r}}_{\text{potential fcn.}} = \vec{F}$.
- conditions equivalent to conservativity, incl. $\operatorname{curl}(\vec{F}) = 0$ when domain simply connected; construction of potential function (solve for f s.t. $f_x = P$, $f_y = Q$, etc.)
- multiple/iterated integrals, applications (volume, centroid, etc.). Good problem to think about : p. 415 #30 — wants volume of $x^2+z^2 \le 1$ intersected with $y^2+z^2 \le 1$.
- change of variable, esp. polar, cylindrical, and spherical coordinates/integration

- surface integrals : $\iint_S f\,dS$, $\underbrace{\iint_S \vec{F}\cdot\hat{n}\,dS}_{\text{flux}}$, $\iint_S P\,dy\wedge dz + \cdots$

- Stokes theorem (= nonplanar Green's thm.):

$$\iint_S (\mathrm{curl}\,\vec{F})\cdot\hat{n}\,dS = \oint_{\partial[S]} \vec{F}\cdot d\vec{r}$$

(So if you are asked to find a line integral around a boundary curve and it looks impossible to do directly...)

- line integrals : $\int_C f\,ds$, $\int_C \vec{F}\cdot d\vec{r}$, $\int_C P\,dx + Q\,dy + \cdots$ (convert to integral on $[a,b]$ by using a parametrization $\vec{r}$; ds becomes $\|\vec{r}\,'(t)\|\,dt$, $d\vec{r}$ becomes $\vec{r}\,'(t)\,dt$, etc.)

- FTC II : $\int_C \vec{\nabla} f\cdot d\vec{r} = f(\vec{b}) - f(\vec{a})$

  $\vec{a} \xrightarrow{C} \vec{b}$

  FTC I : $\vec{F}$ conservative $\Rightarrow \vec{\nabla}\underbrace{\int_{\vec{x}_0}^{\vec{x}} \vec{F}\cdot d\vec{r}}_{\text{potential fcn.}} = \vec{F}$.

- conditions equivalent to conservativity, incl. $\mathrm{curl}(\vec{F}) = 0$ when domain simply connected; construction of potential function (solve for f s.t. $f_x = P$, $f_y = Q$, etc.)

- multiple/iterated integrals, applications (volume, centroid, etc.). Good problem to think about : p. 415 #30 — wants volume of $x^2+z^2 \le 1$ intersected with $y^2+z^2 \le 1$.

- change of variable, esp. polar, cylindrical, and spherical coordinates/integration

- Green's theorem & Gauss's theorem in the plane

- Surface integrals : $\iint_S f\, dS$, $\underbrace{\iint_S \vec{F}\cdot\hat{n}\, dS}_{\text{flux}}$, $\iint_S P\, dy\wedge dz + \cdots$
- Stokes theorem (= nonplanar Green's thm.):

$$\iint_S (\text{curl}\,\vec{F})\cdot\hat{n}\, dS = \oint_{\partial[S]} \vec{F}\cdot d\vec{r}$$

- Gauss theorem

$$\iiint_V \text{div}\,\vec{F}\, dV = \iint_{\partial[V]} \vec{F}\cdot\hat{n}\, dS$$

(So if you are asked to find a "flux of $\vec{F}$ thru S" type surface $\iint$, and it looks impossible to that directly...)

- line integrals : $\int_C f\, ds$, $\int_C \vec{F}\cdot d\vec{r}$, $\int_C P\,dx + Q\,dy + \cdots$ (convert to integral on $[a,b]$ by using a parametrization $\vec{r}$; ds becomes $\|\vec{r}\,'(t)\|dt$, $d\vec{r}$ becomes $\vec{r}\,'(t)\,dt$, etc.)
- FTC II : $\int_C \vec{\nabla} f\cdot d\vec{r} = f(\vec{b}) - f(\vec{a})$ ($\vec{a} \xrightarrow{C} \vec{b}$)

  FTC I : $\vec{F}$ conservative $\Rightarrow \vec{\nabla}\underbrace{\int_{\vec{x}_0}^{\vec{x}} \vec{F}\cdot d\vec{r}}_{\text{potential fcn.}} = \vec{F}$.
- conditions equivalent to conservativity, incl. $\text{curl}(\vec{F}) = 0$ when domain simply connected; construction of potential function (solve for f s.t. $f_x = P$, $f_y = Q$, etc.)
- multiple/iterated integrals, applications (volume, centroid, etc.). Good problem to think about : p. 415 #30 — wants volume of $x^2+z^2 \le 1$ intersected with $y^2+z^2\le 1$.
- change of variable, esp. polar, cylindrical, and spherical coordinates/integration
- Green's theorem & Gauss's theorem in the plane

- surface integrals: $\iint_S f\,dS$, $\underbrace{\iint_S \vec{F}\cdot\hat{n}\,dS}_{\text{flux}}$, $\iint_S P\,dy\wedge dz + \cdots$
- Stokes theorem (= nonplanar Green's thm.):

$$\iint_S (\operatorname{curl}\vec{F})\cdot\hat{n}\,dS = \oint_{\partial[S]} \vec{F}\cdot d\vec{r}$$

- Gauss theorem

$$\iiint_V \operatorname{div}\vec{F}\,dV = \iint_{\partial[V]} \vec{F}\cdot\hat{n}\,dS$$

- gradient = curl 0 ; curl = div 0 (at least, on a convex region)
- line integrals: $\int_C f\,ds$, $\int_C \vec{F}\cdot d\vec{r}$, $\int_C P\,dx + Q\,dy + \cdots$ (convert to integral on $[a,b]$ by using a parametrization $\vec{r}$; ds becomes $\|\vec{r}\,'(t)\|\,dt$, $d\vec{r}$ becomes $\vec{r}\,'(t)\,dt$, etc.)
- FTC II: $\int_C \vec{\nabla} f\cdot d\vec{r} = f(\vec{b}) - f(\vec{a})$ $\quad \vec{a} \xrightarrow{C} \vec{b}$

  FTC I: $\vec{F}$ conservative $\Rightarrow \vec{\nabla}\underbrace{\int_{\vec{x}_0}^{\vec{x}} \vec{F}\cdot d\vec{r}}_{\text{potential fcn.}} = \vec{F}$.
- conditions equivalent to conservativity, incl. $\operatorname{curl}(\vec{F}) = 0$ when domain simply connected; construction of potential function (solve for f s.t. $f_x = P$, $f_y = Q$, etc.)
- multiple/iterated integrals, applications (volume, centroid, etc.). Good problem to think about: p. 415 #30 — wants volume of $x^2+z^2 \le 1$ intersected with $y^2+z^2 \le 1$.
- change of variable, esp. polar, cylindrical, and spherical coordinates/integration
- Green's theorem & Gauss's theorem in the plane