MATH 233 (SPRING 2016)
MIDTERM EXAM 3 REVIEW SUGGESTIONS

This two-hour exam will consist of 14 multiple-choice questions, covering material through Lecture 32. Make sure to bring a number 2 pencil. Calculators and 3×5-cards will not be permitted. Exam scores will be available the following day.

The exam questions are drawn from the following list of skills and topics from §§14.8, 15.1, 15.2, 15.3 15.4, 15.9, and 16.1 in Stewart. Topics not listed will not be covered.

14.8	Lagrange multipliers. Be able to use them to solve max/min problems with 1 or 2 constraints, or on a region with boundary (using Lagrange on boundary).
15.1	Know what a Riemann sum is and how they compare with double integrals. Know midpoint rule. Perform double integrals over rectangles, either by iterated integration or by splitting the integral into a product (if $f(x,y) = F(x)G(y)$).
15.2	Be able to do double integrals over x-simple and y-simple regions D (or to switch order of integration if necessary), and know how to use this in computing areas of regions, volumes of solids, and average value of a function.
15.3	Integrating in polar coordinates, with applications similar to 15.2.
15.4	Use double integrals (in x, y or r, θ) to compute center of mass of a lamina, and to solve basic probability problems (like the one on meeting for coffee we did in class).
15.9	Know how to change variables in double integrals, including how to compute the Jacobian $\frac{\partial(x,y)}{\partial(u,v)}$, and how to choose u and v advantageously (so that the transformed region of integration is "nice").
16.1	Be able to recognize/match vector field and picture. Know what a conservative field is, and what flow lines are.

Practice Midterm 3

Part I: Multiple choice problems

(1) Find the minimum value of $f(x, y, z) = x^2 - 6x + y^2 - 8y + 7$ on the closed disk $x^2 + y^2 \leq 1$.
(A) -10
(B) -6
(C) -2
(D) 0
(E) 9
(F) 18

(2) Let $f(x, y)$ be a function on the rectangle $R = [4, 5] \times [2, 4]$, with values given in the table:

	$x = 4$	$x = 4.5$	$x = 5$
$y = 2$	1	2	3
$y = 3$	8	9	4
$y = 4$	7	6	5

Taking $m = n = 2$ and assuming the maximum and minimum values of f on each subrectangle R_{ij} occur at a vertex, find the Riemann sum value which gives the best (i.e. least) upper bound on $\iint_R f(x,y)dA$.
(A) 36
(B) 30
(C) 24
(D) 18
(E) 15
(F) 12

(3) Determine the volume of the solid under $z = 4xy + x^2$ over the rectangle $R = [1,2] \times [0,3]$.
(A) 34
(B) 41
(C) 46
(D) 51
(E) 61
(F) 68

(4) Evaluate $\int_0^{\sqrt{\ln 2}} \int_0^1 \frac{xy\,e^{x^2}}{1+y^2}\,dy\,dx$.
(A) $\frac{1}{4}\ln 2$
(B) $\frac{1}{2}\ln 2$
(C) $\ln 2$
(D) $\frac{1}{4}$
(E) $\frac{1}{2}$
(F) 1

(5) What is the volume of the solid in the first octant bounded by the cylinder $y = x^2$ and the planes $x = 0$, $z = 0$, and $y + z = 1$?
(A) $\frac{64}{15}$
(B) $\frac{32}{15}$
(C) $\frac{16}{15}$
(D) $\frac{8}{15}$
(E) $\frac{4}{15}$
(F) $\frac{2}{15}$

(6) Compute $\int_0^4 \int_{\sqrt{x}}^2 \cos(y^3)\,dy\,dx$. [Hint: you will have to draw the region of integration to do this one.]
(A) $\frac{1}{3}(1-\cos 8)$
(B) $\frac{1}{3}\sin 8$
(C) $\frac{1}{3}\cos 8$
(D) $3(1-\cos 8)$
(E) $3\sin 8$
(F) $3\cos 8$

(7) Find the volume of a wedge cut from a tall right solid circular cylinder of radius 5 (sitting on the xy plane) by a plane through a diameter of the base and making a $45°$ angle with the base.
(A) $\frac{1}{3}$
(B) $\frac{5}{3}$

(C) $\frac{10}{3}$
(D) $\frac{50}{3}$
(E) $\frac{125}{3}$
(F) $\frac{250}{3}$

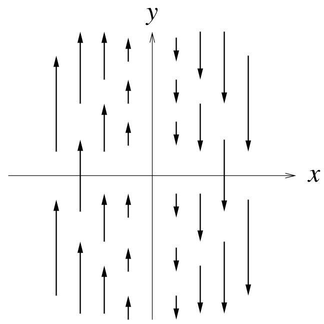
(8) Evaluate $\iint_D \frac{1}{x^2+y^2} dA$, where D is the region between the circles $x^2 + y^2 = 4$ and $x^2 + y^2 = 16$.
(A) $\frac{\pi}{2}$
(B) π
(C) 2π
(D) $\pi \ln 2$
(E) $2\pi \ln 2$
(F) $2\pi \ln 4$

(9) Determine the center of mass of the homogeneous (i.e. of constant density) lamina bounded by the cardioid $r = 1+\sin\theta$. [Hint: you may want to use $2\sin^2\theta = 1-\cos^2 2\theta$ and $2\cos\theta\sin\theta = \sin 2\theta$.]
(A) $(0, \frac{10}{9})$
(B) $(0, \frac{5}{6})$
(C) $(0, 1)$
(D) $(0, \frac{2}{3})$
(E) $(0, \frac{8}{15})$
(F) $(0, \frac{16}{15})$

(10) Let X [resp. Y] denote the number of inches of snow that will fall in St. Louis [resp. Boston] as independent weather systems move through each city. Assume X has probability distribution $F(x) = \begin{cases} \frac{1}{10}, & 0 \le x \le 10 \\ 0, & \text{otherwise} \end{cases}$ and Y has probability distribution $G(y) = \begin{cases} \frac{1}{10}e^{-y/10}, & y \ge 0 \\ 0, & \text{otherwise} \end{cases}$. Calculate the probability that St. Louis gets more snow than Boston, and then decide which approximation (to the nearest whole percentage) is correct:
(A) 1%
(B) 17%
(C) 37%
(D) 51%
(E) 73%
(F) 99%

(11) If we wish to change variables in a double integral from (x, y) to (u, v), where $x = uv$ and $y = u/v$, we must replace $dx\,dy$ by what function times $du\,dv$?
(A) 2
(B) -2
(C) u^2
(D) $-u/v$
(E) $2v/u$
(F) $-2u/v$

(12) Which vector field is sketched in the picture?



(A) $-y\hat{\imath} + x\hat{\jmath}$
(B) $x\hat{\imath}$
(C) $-y\hat{\imath}$
(D) $-x\hat{\jmath}$
(E) $x\hat{\imath} + y\hat{\jmath}$
(F) $y\hat{\jmath}$

Part II: Hand-graded problems

(1) Maximize and minimize $f(x,y,z) = yz + xy$ subject to the two constraints $xy = 1$, $y^2 = 1 - z^2$.

(2) What does it mean for a vector field $\vec{F}$ to be conservative? Is $\vec{F}(x,y) = x\hat{\imath}$ conservative?

(3) Evaluate $\iint_D \exp\left(\frac{x-y}{x+y}\right) dA$, where D is the quadrilateral with vertices at $(1,0)$, $(2,0)$, $(1,1)$, $(\frac{1}{2}, \frac{1}{2})$, by making an appropriate change of variables. [Remark: your answer should be positive.]