## Math 233 Practice Final Exam

### Part I: Multiple choice problems

(1) Find the volume of the parallelepiped with edges $\langle 3,2,1\rangle$, $\langle 1,1,2\rangle$ and $\langle 1,3,3\rangle$.
(A) -11
(B) -7
(C) -3
(D) 1
(E) 5
(F) 9

(2) Consider the curve traced out by $\vec{r}(t) = \langle 8\cos t,\, 6t,\, 8\sin t\rangle$ for $-5 \le t \le 5$. Find the total arclength.
(A) 10
(B) 20
(C) 50
(D) 60
(E) 100
(F) 200

(3) For $\vec{r}(t)$ as in (2), compute the radius of the osculating circle (at any point).
(A) $\frac{25}{2}$
(B) 8
(C) $\frac{5}{4}$
(D) $\frac{4}{5}$
(E) $\frac{1}{8}$
(F) $\frac{2}{25}$

(4) On a distant planet, gravity is $2\,m/s^2$. Determine the speed (in $m/s$) at which a projectile must be thrown at an angle of 30° above the horizontal, from a $10\,m$ high tower, to hit an object on the ground $90\sqrt{3}\,m$ from the base of the tower.
(A) 1
(B) 3
(C) 9
(D) 18
(E) 27
(F) 81

(5) Solid gold is pouring out of a slot machine into a conical pile, in such a way that at a certain instant, the height $h$ is $9\,in$ and increasing at $3\,in/min$, and the radius $r$ is $4\,in$ and increasing at $2\,in/min$. How fast (in $in^3/min$) is the volume increasing at that instant? [Hint: $V = \frac{\pi}{3}r^2h$ for a cone.]
(A) $16\pi$
(B) $32\pi$
(C) $48\pi$
(D) $64\pi$
(E) $80\pi$
(F) $96\pi$

(6) Set $f(x,y) = \frac{2}{9}y^{\frac{3}{2}} + \frac{1}{6}xy$. Find the angle between the $xy$-plane and the tangent plane to $z = f(x,y)$ at $(0, 2, f(0,2))$.
(A) 0
(B) $\frac{\pi}{6}$
(C) $\frac{\pi}{5}$
(D) $\frac{\pi}{4}$
(E) $\frac{\pi}{3}$
(F) $\frac{\pi}{2}$

(7) Compute the directional derivative of $f(x,y,z) = xy + z^2$ at $(1,1,1)$ in the direction *toward* $(5,-3,3)$ from there.
(A) 0
(B) $\frac{1}{3}$
(C) $\frac{2}{3}$
(D) 1
(E) $\frac{4}{3}$
(F) 2

(8) If $T(x,y,z) = 2x^2 + y^2 + z^2$ is the temperature function (in $^\circ C$) on the disk $x^2 + (y-2)^2 + z^2 \leq 9$, what are the hottest and coldest temperatures on the disk?
(A) 24; 0
(B) 25; 0
(C) 26; 0
(D) 24; 1
(E) 25; 1
(F) 26; 1

(9) Find $\iint_{\mathcal{D}} \frac{2}{1+x^2} dA$, where $\mathcal{D}$ is the triangular region with vertices at $(0,0)$, $(1,1)$ and $(0,1)$.
(A) $\frac{\pi}{2}$
(B) $\pi$
(C) $\ln 2$
(D) $2\ln 2$
(E) $\frac{\pi}{2} - 2\ln 2$
(F) $\frac{\pi}{2} - \ln 2$

(10) Consider the disk of radius 1 with center $(0,1)$ and mass density function $\rho(x,y) = \sqrt{x^2+y^2}$. Compute the total mass.
(A) $\frac{32}{9}$
(B) $\frac{16}{3}$
(C) $\frac{8}{3}$
(D) $\frac{4\pi}{3}$
(E) $\pi$
(F) $\frac{2\pi}{3}$

(11) Say we are integrating in $x$ and $y$ and we want to integrate in $u = \frac{x^2}{y}$ and $v = \frac{x^5}{y^2}$. We must replace $dx\,dy$ by what function times $du\,dv$?
(A) $\frac{5v^2}{u^8}$
(B) $\frac{v^2}{u^8}$
(C) $\frac{5x^6}{y^4}$
(D) $\frac{x^6}{y^4}$
(E) $\frac{5u^8}{v^2}$
(F) $\frac{u^8}{v^2}$

(12) Let $\vec{F} = (2x+y)\hat{\imath} + (x-2y)\hat{\jmath}$. Compute $\int_{\mathcal{C}} \vec{F} \cdot d\vec{r}$, where $\mathcal{C}$ is any oriented curve starting at $A = (1,2)$ and ending at $B = (3,0)$.
(A) $-10$
(B) $-5$
(C) 0
(D) 5
(E) 10
(F) the integral is not independent of path

**Part II: Free-Response problems.**

(1) Find the work done by the force field $\vec{F}(x,y,z) = y\hat{\imath} + z\hat{\jmath} + x\hat{k}$ in moving a particle along the oriented curve $\mathcal{C}$ traced out by $\vec{r}(t) = \langle t, t^2, t^3 \rangle$, $t \in [0,1]$.

(2) Are the integrals $\oint_{\mathcal{C}} \vec{F} \cdot d\vec{r}$ of $\vec{F}(x,y,z) = (2xyz + z^2)\hat{\imath} + (x^2 z + z^3)\hat{\jmath} + (x^2 y + 3yz^2)\hat{k}$ around any closed path equal to zero? Why or why not?

(3) Use Gauss's theorem in the plane to compute the flux of $\vec{F}(x,y) = (e^{-y^2} + 2x)\hat{\imath} + (e^{-2x^2} + y)\hat{\jmath}$ across the (counterclockwise oriented) boundary of the triangle with vertices $(0,0)$, $(1,0)$, and $(0,1)$.

(4) Determine a formula for the surface area of the "polar cap" on a sphere of radius $a$ determined by the spherical angle $\alpha$. (For full credit you must compute the integral; of course, the final expression should involve $a$ and $\alpha$.)