MATH 233 LECTURE 12 (§14.1): MULTIVARIABLE FUNCTIONS

A (scalar-valued) function of n variables is a rule assigning a single number to an n-tuple of numbers, viz.

$$\begin{aligned} f: \quad \mathbb{R}^n &\longrightarrow \quad \mathbb{R} \\ (x_1, \ldots, x_n) &\longmapsto \underbrace{f(x_1, \ldots, x_n)}_{\text{number}} \end{aligned}$$

(This is basically the opposite of what we have been doing in Chapter 13.) For us, n will be 2 or 3, and in this lecture $n = 2$.

- That is, we want to discuss functions $f(x, y)$ of two variables, and their graphs $z = f(x, y)$ in 3-dimensional space.
- Level curves: these are the solution sets of equations $f(x, y) = k$ in the xy-plane, for k a constant. They aid in visualizing the graph.
- Given a function $f(x, y)$, you should also consider where (in the xy-plane) it is defined. The domain $Dom(f)$ may not be all of $\mathbb{R}^2$.

Graphing functions of 2 variables.

- Linear functions: $f(x, y) = ax + by + c$. The graph is a plane. Draw it by looking for the x-, y-, and z-intercepts (where $z = f(x, y)$ intersects the axes).
- Quadratic functions: $f(x, y) = ax^2 + by^2 + cxy + dx + ey + f$. The graph is an elliptic or hyperbolic paraboloid.
- There are various other examples that yield quadric surfaces (or part of one), like $f(x, y) = \sqrt{(x - a)^2 + (y - b)^2}$.
- We will look at a couple other examples, including a radially symmetric one.