MATH 233 LECTURE 22 (§14.8): LAGRANGE MULTIPLIERS (CONT'D.)

- Recall: these give a way of maximizing or minimizing a function $f(x_1, \ldots, x_n)$ (of n variables) subject to the constraint $g(x_1, \ldots, x_n) = 0$, by solving the system of $n+1$ equations given by $g = 0$ and $\vec{\nabla} f = \lambda \vec{\nabla} g$. This does away with the need to parametrize the level set $g = 0$.
- Note that this can be used to find the max/min of a function on the boundary of a region S (as part of an unconstrained extremum problem), when that boundary takes the form $g = 0$.
- Sometimes reality imposes more than one constraint on your variables. The simplest case is when you have a function of 3 variables x, y, z, and we want to maximize or minimize $f(x, y, z)$ subject to $g(x, y, z) = 0$ *and* $h(x, y, z) = 0$. The latter two conditions define a curve C as the intersection of 2 surfaces S_1 and S_2in $\mathbb{R}^3$ (defined by $g = 0$ resp. $h = 0$).
- :Lagrange multipliers can handle this situation too. The general form of a vector normal to C is a linear combination of vectors normal to S_1 and S_2, which is to say $\vec{\nabla} g$ and $\vec{\nabla} h$. For the same reasons as before, if f is maximized at a point (x_0, y_0, z_0) on C, then $(\vec{\nabla} f)(x_0, y_0, z_0)$ must be normal to C. So you conclude that we must have

  $$\vec{\nabla} f = \lambda \vec{\nabla} g + \mu \vec{\nabla} h$$

  at a maximum or minimum. (Here λ and μ are the "Lagrange multipliers".) Together with $g = 0$ and $h = 0$, this gives a system of 5 equations in 5 variables (i.e. x, y, z, λ, μ).
- The case of k constraints in n variables is a straightforward generalization: there will be k multipliers. But we won't do more than 2 constraints.