MATH 233 LECTURE 26: MORE ON ITERATED INTEGRALS

The lecture consists mainly of worked examples. I will only summarize here.

- If $R = [1,2] \times [0,\pi]$, then Fubini's theorem gives

  $$\iint_R y\sin(xy)dA = \int_1^2 \left(\int_0^\pi y\sin(xy)dy\right) dx = \int_0^\pi \left(\int_1^2 y\sin(xy)dx\right) dy.$$

  It turns out that while you can reach the answer (which is 0) either way, the second way is much easier.

- To find the volume of the solid bounded by the surface $z = \frac{y}{(xy+1)^2}$ and the planes $z = 0$, $x = 0$, $x = 1$, $y = 0$, $y = 1$, compute the integral $\iint_R \frac{y}{(xy+1)^2} dA$, where $R = [0,1] \times [0,1]$. Again, there is one way of iterating the integral which is much easier to carry out.

- When $f(x,y) = F(x)G(y)$, and $R = [a,b]\times[c,d]$, $\iint_R f(x,y)dA = \left(\int_a^b F(x)dx\right)\left(\int_c^d G(y)dy\right)$. We can use this to integrate functions like $xye^{x^2+y^2}$.

- By Fubini's theorem, $\int_a^b \int_c^d f(x,y)dy\,dx = \int_c^d \int_a^b f(x,y)dx\,dy$ — that is, you can reverse the order of integration. If you are given an iterated integral that looks impossible, sometimes reversing the order makes it possible, for example if the function is $\frac{8x}{(x^2+y^2+1)^2}$.