MATH 233 LECTURE 6 (CHAPTER 10): PLANE CURVES

The material from Chapter 10 you are responsible for is fairly limited, and basically as described in this lecture. Main point: get hands on some curves in 2-D before we go on to curves in 3-D.

Cartesian representation.

- This means that the curve is presented as the solution set of an equation $F(x, y) = 0$ in two variables.

Parametric representation.

- Given by $x = f(t)$, $y = g(t)$; think of this as the motion of a particle on the curve in time.
- Familiar example: $f(t) = x_0 + at$, $g(t) = y_0 + bt$ traces out a line (with direction vector $\langle a, b\rangle$, through (x_0, y_0)).
- Two methods for drawing/understanding curves given in parametric form: (i) plot points at a few values of t; (ii) eliminate the parameter t (to get a Cartesian representation of the curve).
- Basic example of (ii): given $x = f(t) = a\cos t$, $y = g(t) = b\sin t$, write $(x/a)^2 + (y/b)^2 = \cos^2 t + \sin^2 t = 1$ (equation of an ellipse). More examples in class.

Polar representation.

- Given by $r = G(\theta)$, e.g. G constant gives a circle centered at the origin. (More examples in class.)
- Relation between polar and Cartesian coordinates is given by $r = \sqrt{x^2 + y^2}$, $\theta = \arctan(y/x)$, and $x = r\cos\theta$, $y = r\sin\theta$. Use these formulas to convert from polar to Cartesian representation (and vice versa).