MATH 233 LECTURE 7 (§13.1): VECTOR-VALUED FUNCTIONS

- The vector-valued functions we will consider have domain in $\mathbb{R}$ and range in $\mathbb{R}^3$: that is, they take the form

$$\vec{r}(t) = \langle f(t), g(t), h(t) \rangle.$$

  Unless f, g, h are constant, this is really just a way to repackage the parametric equations of a curve.

- Limits and continuity: $\lim_{t\to a} \vec{r}(t) := \langle \lim_{t\to a} f(t), \lim_{t\to a} g(t), \lim_{t\to a} h(t) \rangle$; and $\vec{r}(t)$ is continuous at a if $\lim_{t\to a} \vec{r}(t) = \vec{r}(a)$.
- Provided limits of $\vec{v}(t)$ and $\vec{u}(t)$ at a exist, $\lim_{t\to a} \vec{u}(t) \cdot \vec{v}(t) = \lim_{t\to a} \vec{u}(t) \cdot \lim_{t\to a} \vec{v}(t)$ (and similarly for cross-product).
- Derivatives: $\vec{r}'(t) := \lim_{h\to 0} \frac{\vec{r}(t+h) - \vec{r}(t)}{h} = \langle f'(t), g'(t), h'(t) \rangle$ (provided f, g, h differentiable)
- Key skill: writing down vector-valued functions that "represent" (i.e. parametrize) a given space curve, which might for example be presented as the intersection of two surfaces.