318 PROBLEM SET 8: SUPPLEMENTARY PROBLEM

(a) Show that

$$f\left(\begin{smallmatrix} x \\ y \end{smallmatrix}\right) := \begin{pmatrix} \sqrt{1+\frac{y^2}{2}} \\ \sqrt{2+\frac{x^2}{2}} \end{pmatrix}$$

is a contraction mapping on any rectangle $\mathscr{R} := [-R, R] \times [-R, R]$ with $R > 0$ sufficiently large. Deduce that, as a map from $\mathbb{R}^2$ to $\mathbb{R}^2$, f has a unique fixed point.

(b) Find this fixed point.