Problem set 9

1. Find the fundamental units of $\mathbb{Q}(\sqrt{d})$ (that is, of its ring of integers) for $d = 7, 30,$ and $53$.

2. Use a unit in $\mathbb{Z}[\sqrt{30}]$ to prove that the difference between $241/44$ and $\sqrt{30}$ is less than $5 \times 10^{-5}$.

3. Let $n \in \mathbb{Z}$, $n > 2$ and put $d = n^2 - 2$. Show that $n^2 - 1 + n\sqrt{d}$ is a unit of $\mathbb{Z}[\sqrt{d}]$. Is it necessarily the fundamental unit? (Give a proof or a counterexample.)

4. Give formulae for all the solutions $(x, y) \in \mathbb{Z} \times \mathbb{Z}$ (if any) to $x^2 - 6y^2 = 1$.

5. Give formulae for all the solutions $(x, y) \in \mathbb{Z} \times \mathbb{Z}$ (if any) to $9x^2 - 7y^2 = 1$. 