In addition to these problems, read the introduction to Chapter 4 in Jacobson.

(1) [Jacobson p. 215 #2] Determine \([\mathbb{Q}(\sqrt{2}, \sqrt{3}):\mathbb{Q}]\).

(2) [Jacobson p. 215 #4] Let \(w = \cos(\pi/6) + i \sin(\pi/6)\) (in \(\mathbb{C}\)). Note that \(w^{12} = 1\) but \(w^r \neq 1\) if \(1 \leq r < 12\) (so \(w\) is a generator of the cyclic group of 12th roots of 1).
Show that \([\mathbb{Q}(w):\mathbb{Q}] = 4\) and determine the minimal polynomial of \(w\) over \(\mathbb{Q}\).

(3) [Jacobson p. 215 #6] Let \(E_i, i = 1, 2,\) be a subfield of \(K/F\) such that \([E_i:F] < \infty\).
Show that if \(E\) is the subfield of \(K\) generated by \(E_1\) and \(E_2\) then \([E:F] \leq [E_1:F][E_2:F]\).

(4) [Jacobson p. 215 #8] Let \(E = F(u), u\) transcendental, and let \(K \neq F\) be a subfield of \(E/F\). Show that \(u\) is algebraic over \(K\).

(5) Given that \((\ell, 0)\) is constructible \((\ell \in \mathbb{R}_+),\) show how to construct \((\sqrt{\ell}, 0)\) and \((\ell^2, 0)\). (You must give more detail than Jacobson.)

(6) Construct a regular pentagon “with straightedge and compass”.

(7) Suppose that \(M/L\) and \(L/K\) are extensions, and that \(\alpha \in M\) is algebraic over \(K\). Does \([L(\alpha): L]\) always divide \([K(\alpha): K]\)?