Problem Set 2

[J]=Jacobson

(1) Show that an algebraically closed field must be infinite.
(2) Suppose that $K(\alpha)/K$ is a simple extension and that α is transcendental over K. Show that $K(\alpha)$ is not algebraically closed.
(3) [J] p. 234 #2
(4) [J] p. 234 #4
(5) Let p be a prime number. By factorizing $x^{p-1} - 1$ over $\mathbb{Z}_p$, prove *Wilson's theorem*: i.e., show that $(p-1)! \underset{(p)}{\equiv} -1$.
(6) Let K be a field of positive characteristic.
(i) Show that K is perfect if and only if the Frobenius homomorphism is an automorphism.
(ii) If L/K is a totally inseparable extension (i.e. every element of $L \backslash K$ is inseparable), show that the minimal polynomial of any element of L over K is of the form $x^{p^n} - \alpha$, where $\alpha \in K$.
(7) Suppose that L/K is algebraic. Show that there is a greatest intermediate field $M(\subset L)$ such that M/K is normal.
(8) Suppose that L/K is finite, with normal closure L^c/L. Show that L/K is separable if and only if there are exactly $[L : K]$ embeddings of L into L^c fixing K.