

PROBLEM SET 4

[J]=Jacobson

- (1) Factorize $x^{p^p} - x$ over \mathbb{Z}_p .
- (2) Let $p < q$ be primes, $p \nmid q - 1$. Show that there is an extension L/\mathbb{Z}_q which is a splitting field extension for each of the polynomials $x^p - a$ ($a \in \mathbb{Z}_q^*$).
- (3) Suppose that L/K is finite and separable and M/L is finite simple. Show that M/K is simple.
- (4) Show that the simple transcendental extension $K(t)/K$ has infinitely many intermediate fields.
- (5) [J] p. 250 #6
- (6) [J] p. 251 #10
- (7) [J] p. 151 #19 (on Möbius function)
- (8) [J] p. 295 #2